(Jntuh)

RADAR SYSTEMS
(EC 812 PE)
(ELECTIVE V)
UNIT – 1 E
B.TECH IV YEAR II SEMESTER

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BIET

(Jntuh)

Acknowledgements

The contents , figures , graphs etc., are taken from the following Text book & others

“ INTRODUCTION TO RADAR SYSTEMS “

Merill I.Skolnik

Second Edition

Tata Mcgraw – Hill publishing company

Special indian edition

NECESSITY FOR PROBABILITY DENSITY FUNCTIONS

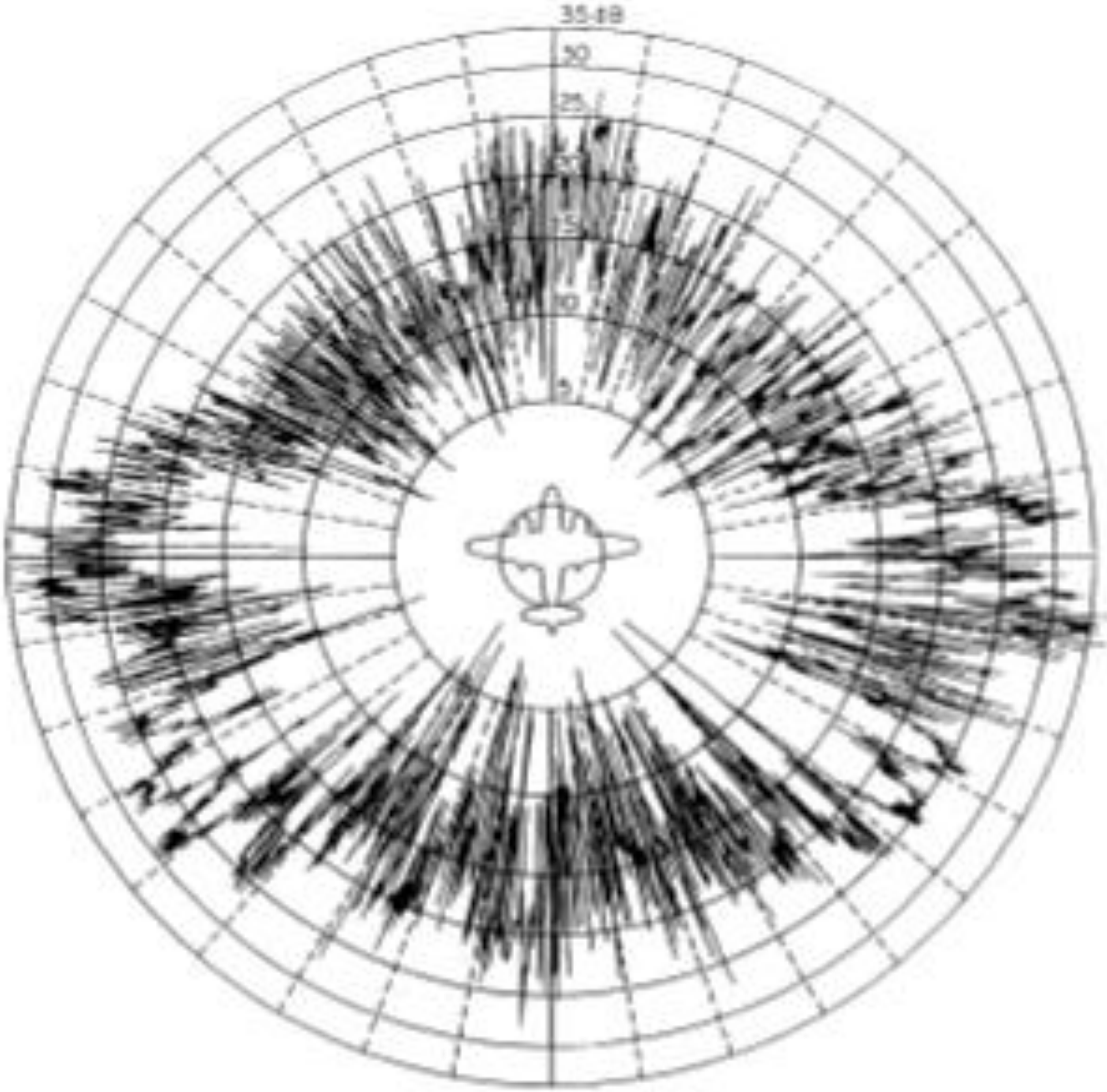
NECESSITY FOR PROBABILISTIC TERMS

- $$R_{max}^{4} = \frac{P_T \; G \; A_e \; \sigma}{(4\,\pi)^2 \; K \, T_O \; B_n \; F_n \; (S/N)_{min}}$$

- In the above equation ‘N’ Noise is a random phenomenon which means that it does not have a fixed value at any instant. It is fluctuating. So $(S/N)_{min}$ keeps fluctuating.

NECESSITY FOR PROBABILISTIC TERMS (CONTD…)

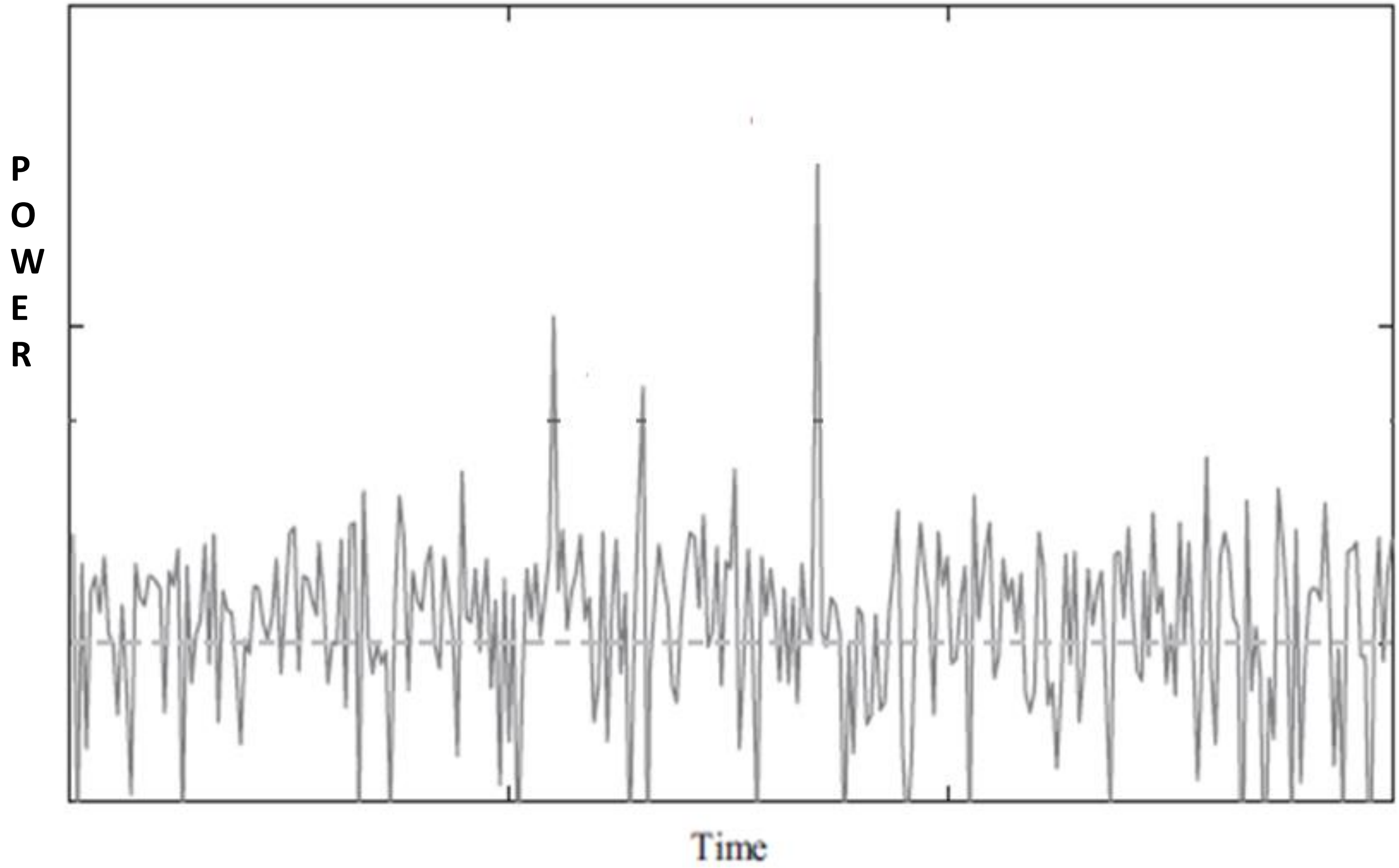
- Similarly ‘ σ ‘ Radar cross section is also a fluctuating quantity.
- Because $(S/N)_{min}$ and σ are fluctuating , detection of signals becomes a random phenomenon.
- Random values are described by probabilistic terms.



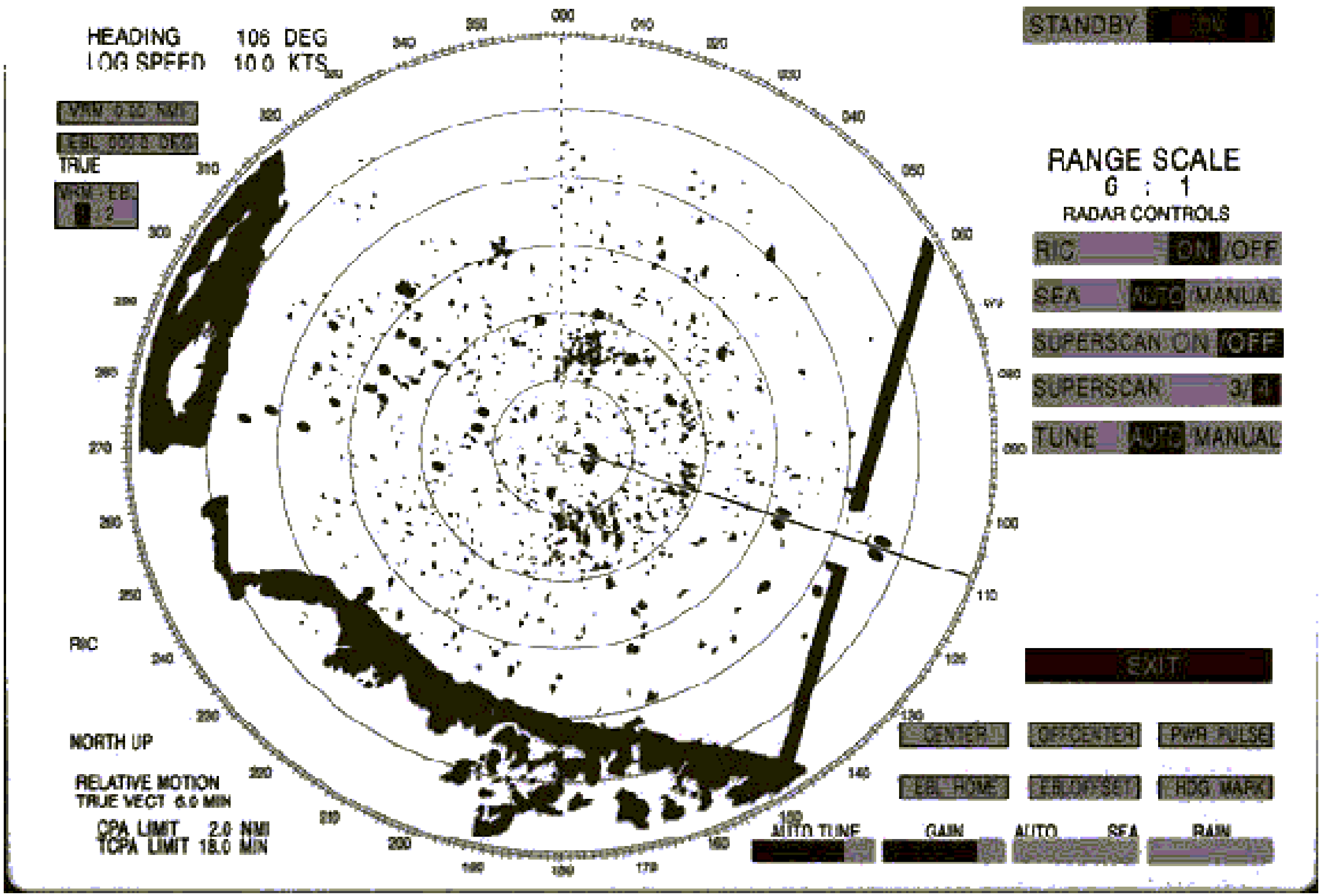
RCS (σ) OF AN AIRCRAFT

NOISE

- NOISE



RADAR DISPLAY



PROBABILITY DENSITY FUNCTIONS

PROBABILITY

- (Jntuh) **Discuss about probability density functions**
- **Probability:** is a measure of the likelihood of the occurrence of an event. Its value lies between

  0 to 1
- **Probability Density Function:**
- p(x) = $$\lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{(\text{Number of values within } \Delta x)/_{\Delta x}}{\text{total number of values} = N}$$

Where x = random quantity say noise voltage

PROBABILITY (CONTD…)

- Let ‘x’ represents Noise voltage
- Probability that a particular value ‘x’ lies within infinitesimal interval ‘dx’ centered at ‘x‘ = p(x) dx

- Probability (x_1< ‘x’ < x_2) = $\int_{x1}^{x2} p(x)\, dx$

- $\int_{-\infty}^{+\infty} p(x)\, dx = 1$

PROBABILITY (CONTD…)

- Mean (Average) $(x)_{av}$ = m_1 = $\int_{-\infty}^{+\infty} x\, p(x)\, dx$

- Mean square value $(x^2)_{av}$ = m_2 = $\int_{-\infty}^{+\infty} x^2\ p(x)\, dx$

- m_1 = first moment of random variable x
  = d.c. component
- m_2 = second moment
- $m_2 \times$ resistance = Average Power (when ‘x’ represents ‘current’)

PROBABILITY (CONTD…)

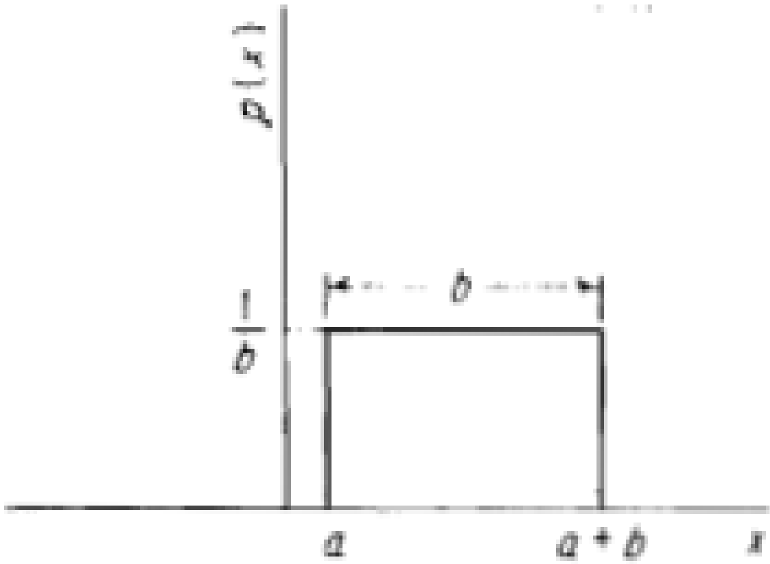
- Variance: σ^2 (not the radar cross section) is the mean square deviation of 'x' about its mean m_1

$$\sigma^2 = \langle (x - m_1)^2 \rangle_{av} = \int_{-\infty}^{\infty} (x - m_1)^2 \, p(x)\, dx$$

- Standard Deviation: σ is the square root of variance also is the rms (root mean square) value of the a-c component.

PROBABILITY DENSITY FUNCTIONS

- Uniform Distribution

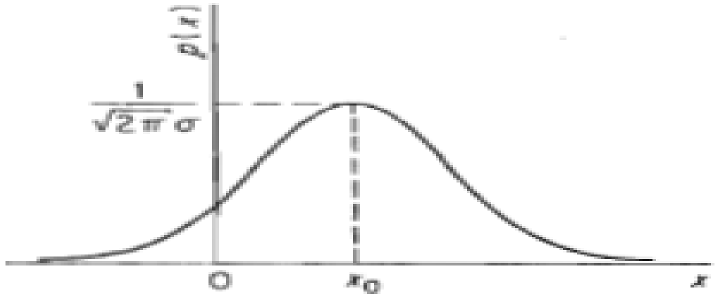


p(x) = 1/b for a < ‘x’ < a+b

= 0 for ‘x’ < a and ‘x’ > a+b

PROBABILITY DENSITY FUNCTIONS (CONTD…)

- **Gaussian or Normal Distribution**



$$p(x) = \frac{1}{\sqrt{2\ \pi\ \sigma^2}}\ \exp\ \frac{-(x - x_0)^2}{2\ \sigma^2}$$

'x' = Variable

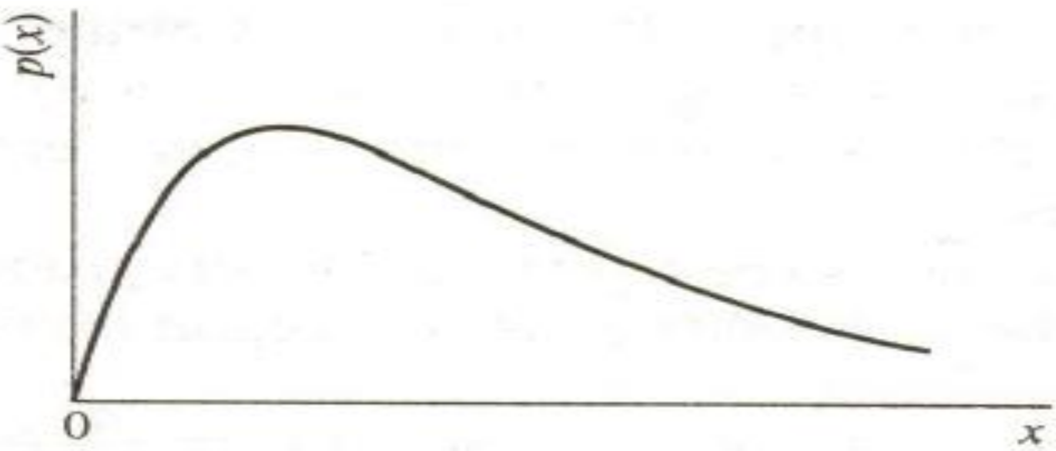
$\boldsymbol{\sigma^2}$ = variance = mean square deviation of 'x' about its mean = Ψ

x_0 = Mean 'x' = average 'x'

- Many sources of noise like Thermal Noise or shot noise are represented by Gaussian Distribution.

PROBABILITY DENSITY FUNCTIONS (CONTD…)

- **Raleigh Distribution**



$$\mathbf{p(x) = \frac{2\ x}{m_2}\ exp\ \left(-\frac{x^2}{m_2}\right)}$$

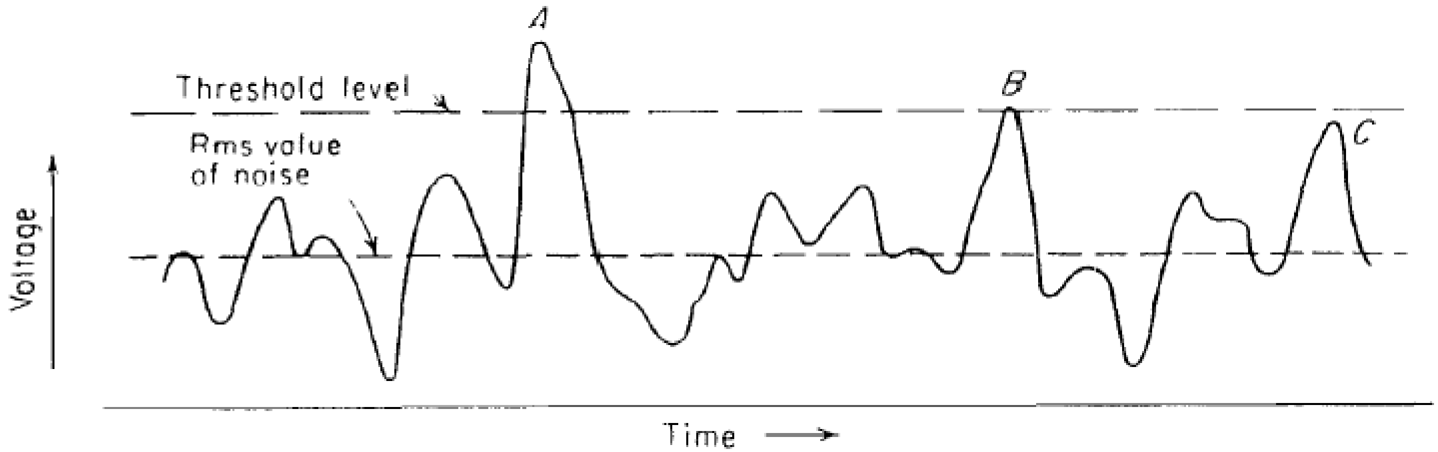
- Where $m_2 = (x^2)_{av}$ is the mean square value of x

PROBABILITY OF DETECTION AND PROBABILITY OF FALSE ALARM

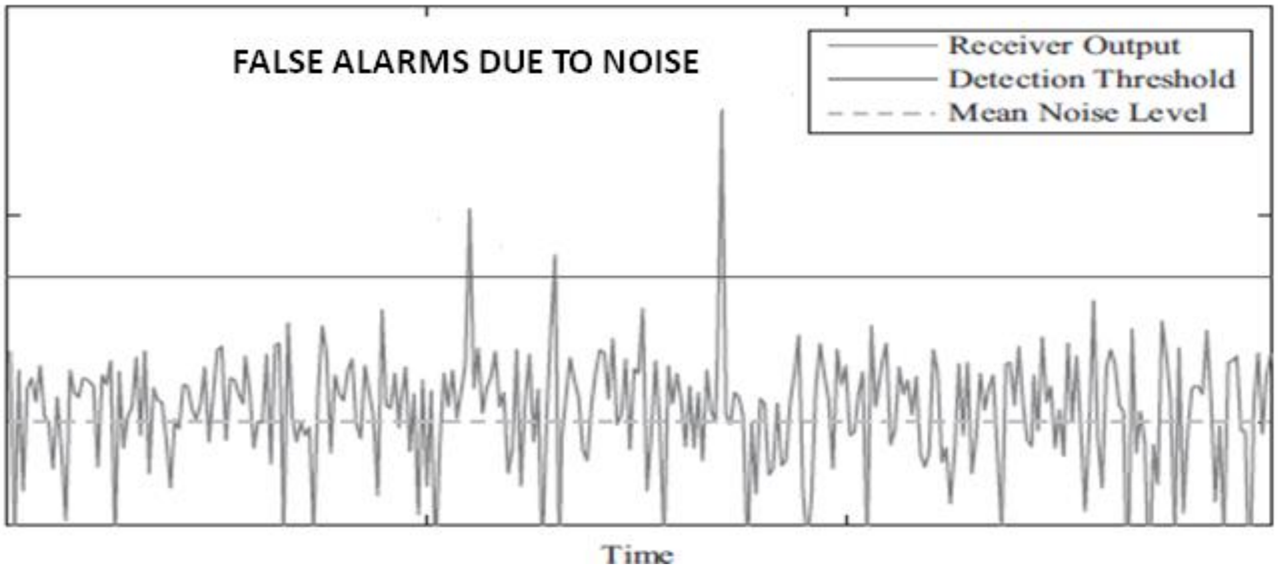
PROBABILITY (CONTD…)

- (Jntuh) What is False alarm, Probability of detection and False alarm time
- Parameters for design:
- 1. Probability of detection
  2. Probability of false alarm
  3. False alarm time

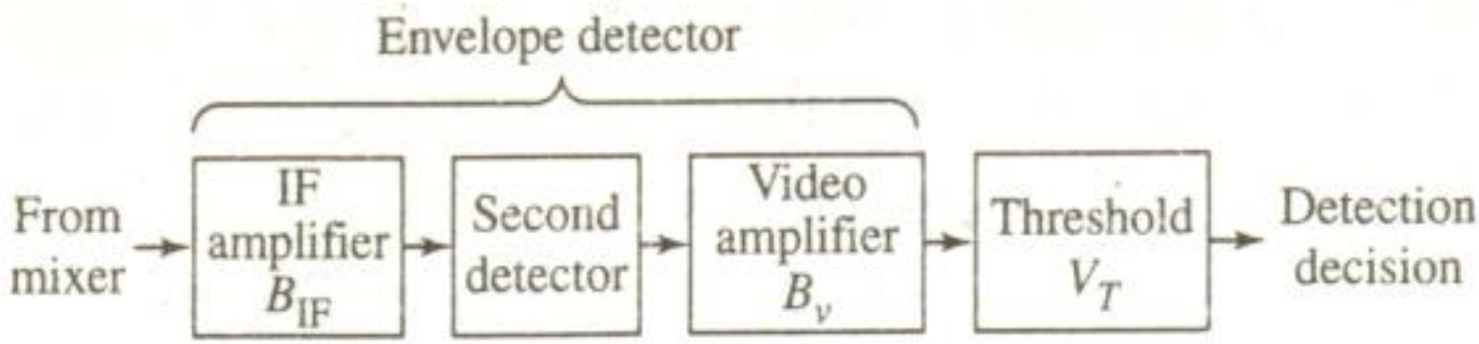
PROBABILITY OF DETECTION



PROBABILITY OF FALSE ALARM

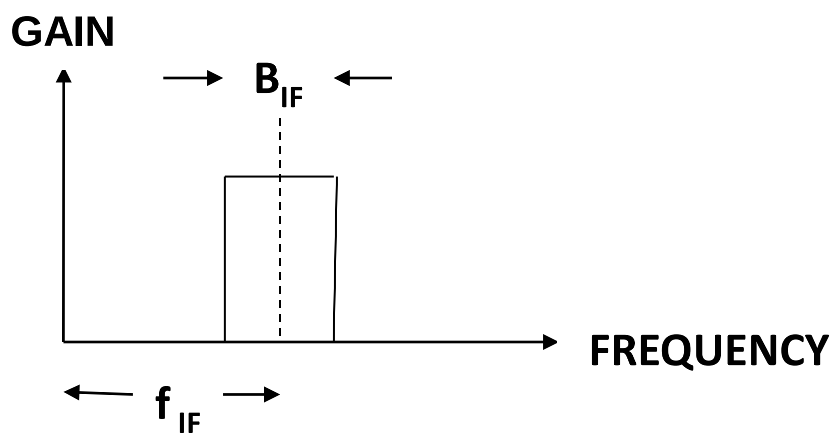


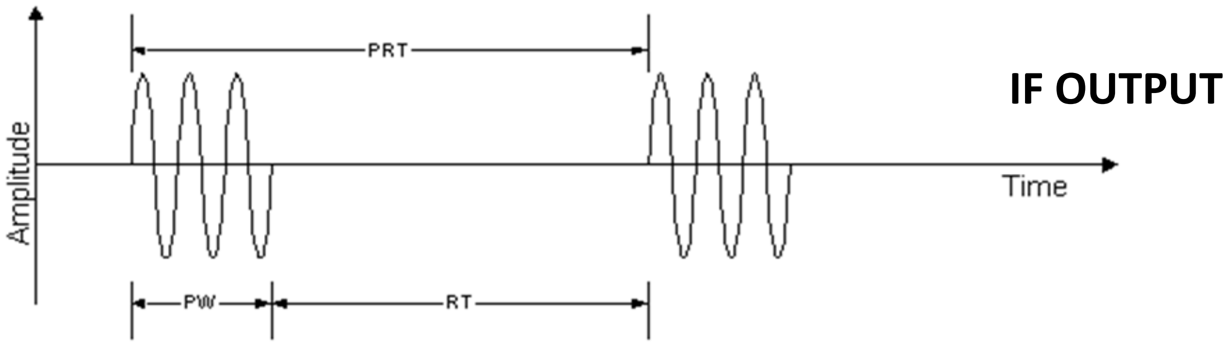
ENVELOPE DETECTOR

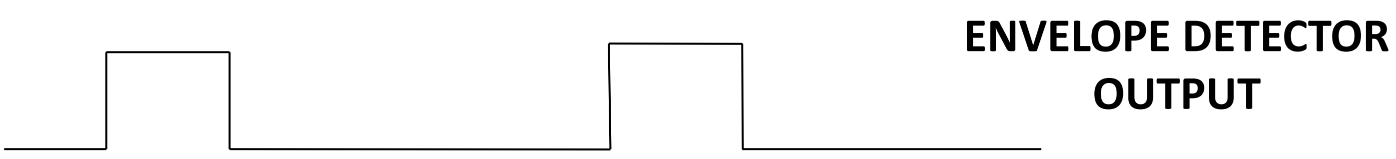


- Envelope Detector passes the modulation and rejects the carrier
- B_v Video Band width
- B_{IF} = IF Amplifier Band width

ENVELOPE DETECTOR (CONTD..)

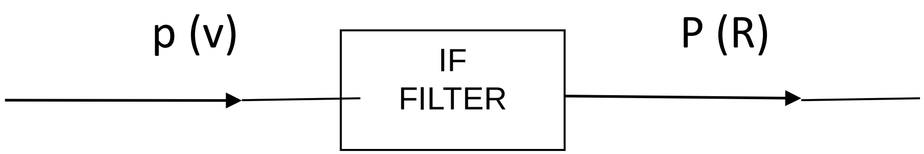


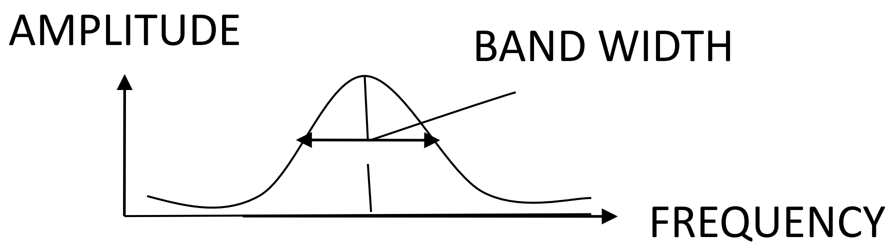




PROBABILITY OF FALSE ALARM

- Probability of False Alarm = Probability that noise crosses threshold and declared as a target





v = noise voltage

R = envelope

Ψo = mean square value of noise voltage =

If Gaussian noise with pdf p(v) is passed through a narrow band filter whose Bandwidth is small compared to mid frequency $f_{o,}$ output p(R) is shown to be Rayleigh pdf by 'RICE'

PROBABILITY OF FALSE ALARM (CONTD…)

- $p(v) = \frac{1}{\sqrt{2 \pi \varphi_0}} \exp \frac{-v^2}{2 \varphi_0}$

v = noise voltage, φ_0 = variance= σ^2(mean square value)

Mean value of v is taken as zero

$$p(R) = \frac{R}{\varphi_0} \exp\left(\frac{-R^2}{2 \varphi_0}\right)$$

R= amplitude of IF filter output

- Probability ‘R’ lies between v_1 and v_2 is given by

$$\int_{V1}^{V2} \frac{R}{\varphi_0} \exp\left(\frac{-R^2}{2 \varphi_0}\right)$$

PROBABILITY OF FALSE ALARM (CONTD…)

- Probability that noise voltage 'R' exceeds voltage Threshold V_T is

  Probability (v_T<R< ∞) =

$$\int_{VT}^{\infty} \frac{R}{\varphi_0} \exp\left(\frac{-R^2}{2\ \varphi_0}\right)\ dR$$

$$= \frac{R}{\varphi_0} \int_{V_T}^{\infty} \exp\left(\frac{-R^2}{2\ \varphi_0}\right) \frac{d(\frac{-R^2}{2\ \varphi_0})}{\frac{-\ 2\ R}{2\ \varphi_0}}$$

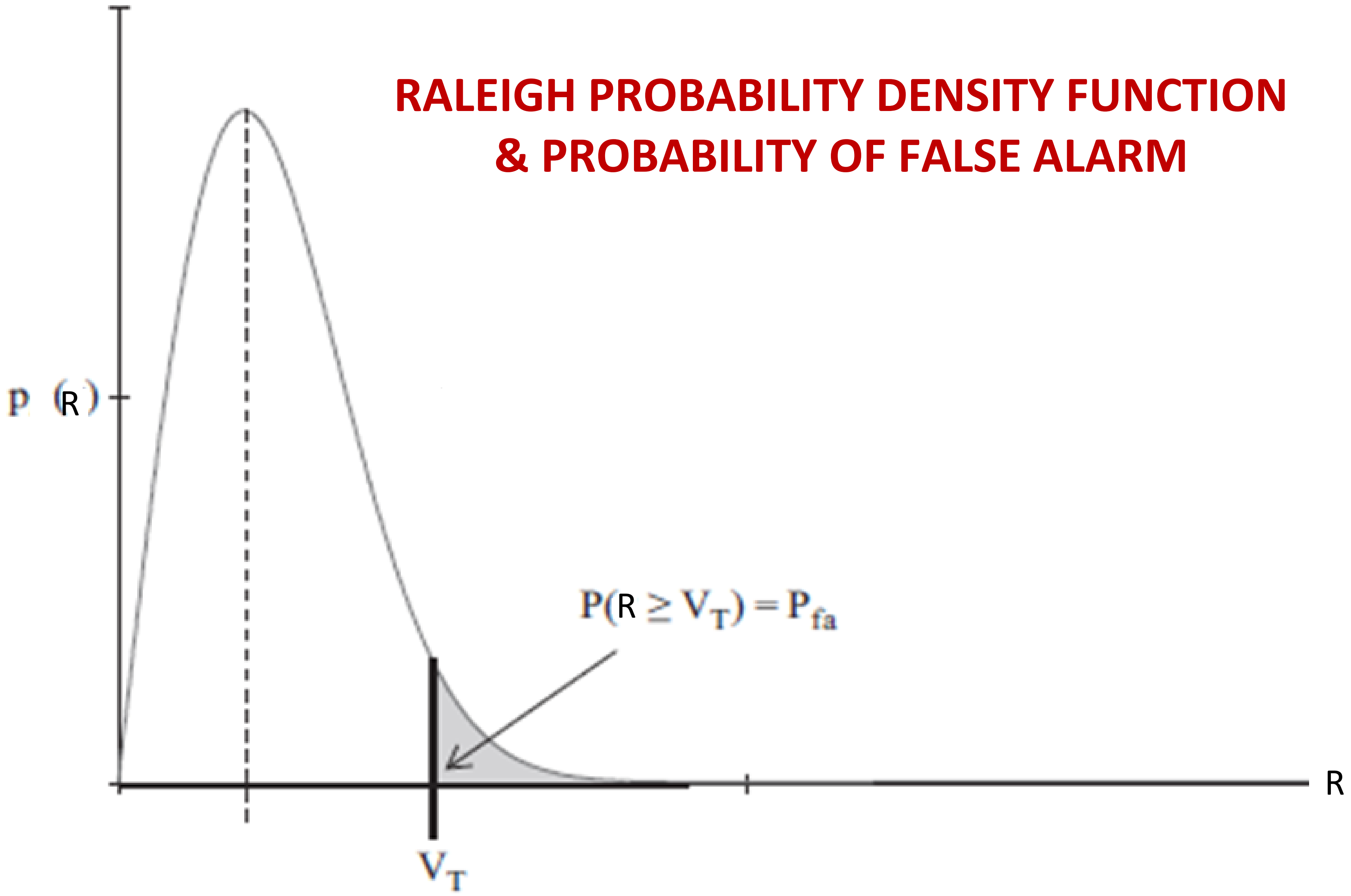
- $$\left(-\exp\frac{-R^2}{2\ \varphi_0}\right)_{V_T}^{\infty} = -\exp\frac{-\infty^2}{2\ \varphi_0} + \exp\frac{-{V_T}^2}{2\ \varphi_0}$$

PROBABILITY OF FALSE ALARM (CONTD…)

- Probability that noise voltage 'R' exceeds voltage Threshold V_T is P_{fa}
- $P_{fa} = -\exp\frac{-\infty^2}{2\ \varphi_0} + \exp\frac{-V_T{}^2}{2\ \varphi_0}$
- $\exp^{-\infty^2} = \frac{1}{\exp^{\infty^2}} = \frac{1}{\infty} = 0$
- Probability that Noise crosses Threshold V_T =

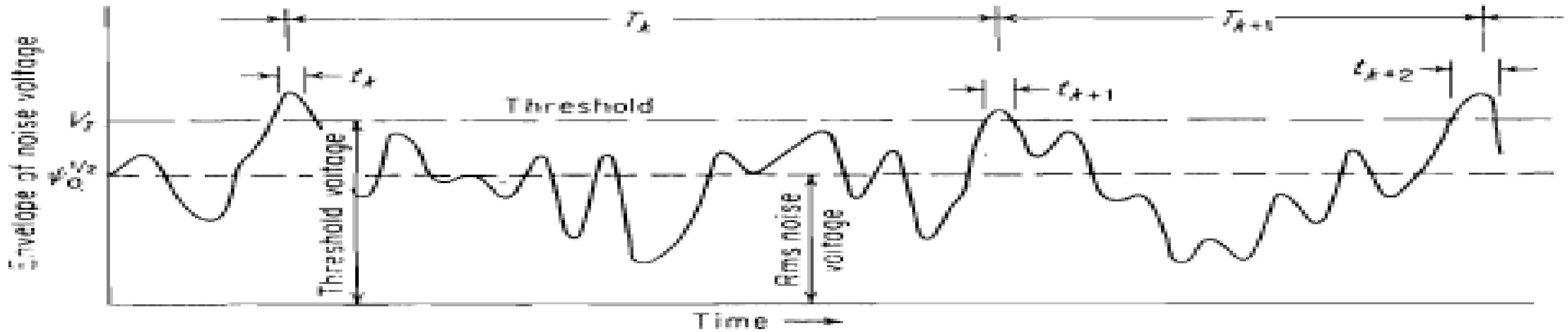
  Probability of False Alarm = P_{fa}
- $P_{fa} = \exp\frac{-V_T{}^2}{2\ \varphi_0}$

RALEIGH PROBABILITY DENSITY FUNCTION & PROBABILITY OF FALSE ALARM



FALSE ALARM TIME

- **False Alarm Time:** is defined as the average time interval between crossings of the Threshold (when
- slope of crossings is positive) = T_{fa}



Envelope of RX with noise alone

- $$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{K=1}^{N} T_K = \left[\frac{T_K + T_{K+1} + T_{K+2} + \ldots}{N}\right]$$

where T_k = Time between crossings of Threshold V_T

FALSE ALARM TIME (CONTD…)

- False Alarm Probability = P_{fa}

- $P_{fa} = \dfrac{\text{Duration of time envelope above threshold}}{\text{Total time it could have been above threshold}}$

- $P_{fa} = \dfrac{\sum_{K=1}^{N} t_K}{\sum_{K=1}^{N} T_K} = \dfrac{(t_K)_{av}}{(T_K)_{av}}$

$$= \frac{\text{Average duration of Noise pulse}}{T_{fa}}$$

t_k = average duration of a noise Pulse = $\dfrac{1}{B_{IF}}$

B_{IF} = Bandwidth of IF Amplifier

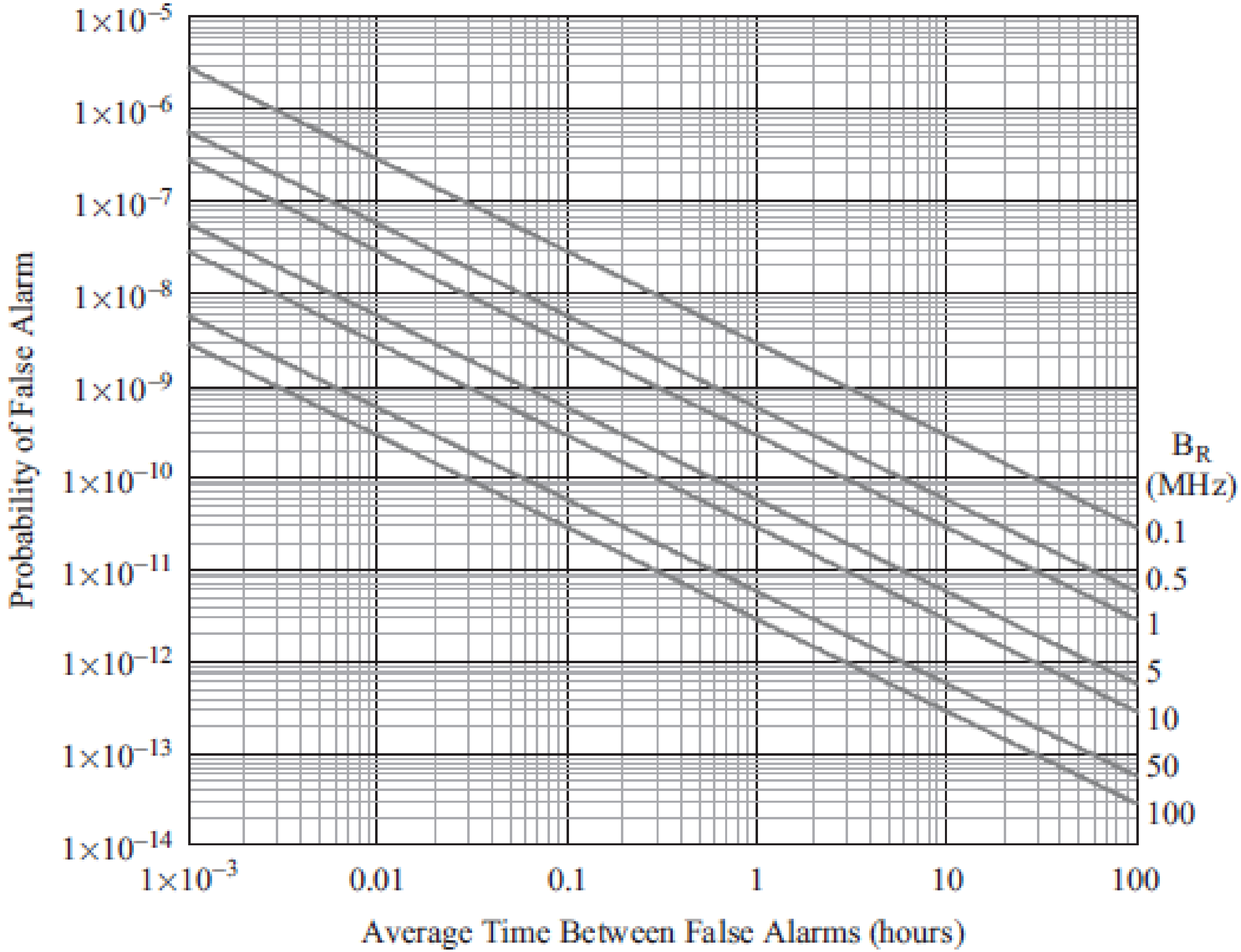
FALSE ALARM TIME (CONTD…)

- $P_{fa} = \dfrac{1}{T_{fa}\ B_{IF}}$
- But from earlier derivation $P_{fa} = \exp\left[\dfrac{-{V_T}^2}{2\ \varphi_0}\right]$
- $P_{fa} = \dfrac{1}{T_{fa}\ B_{IF}} = \exp\left[\dfrac{-{V_T}^2}{2\ \varphi_0}\right]$

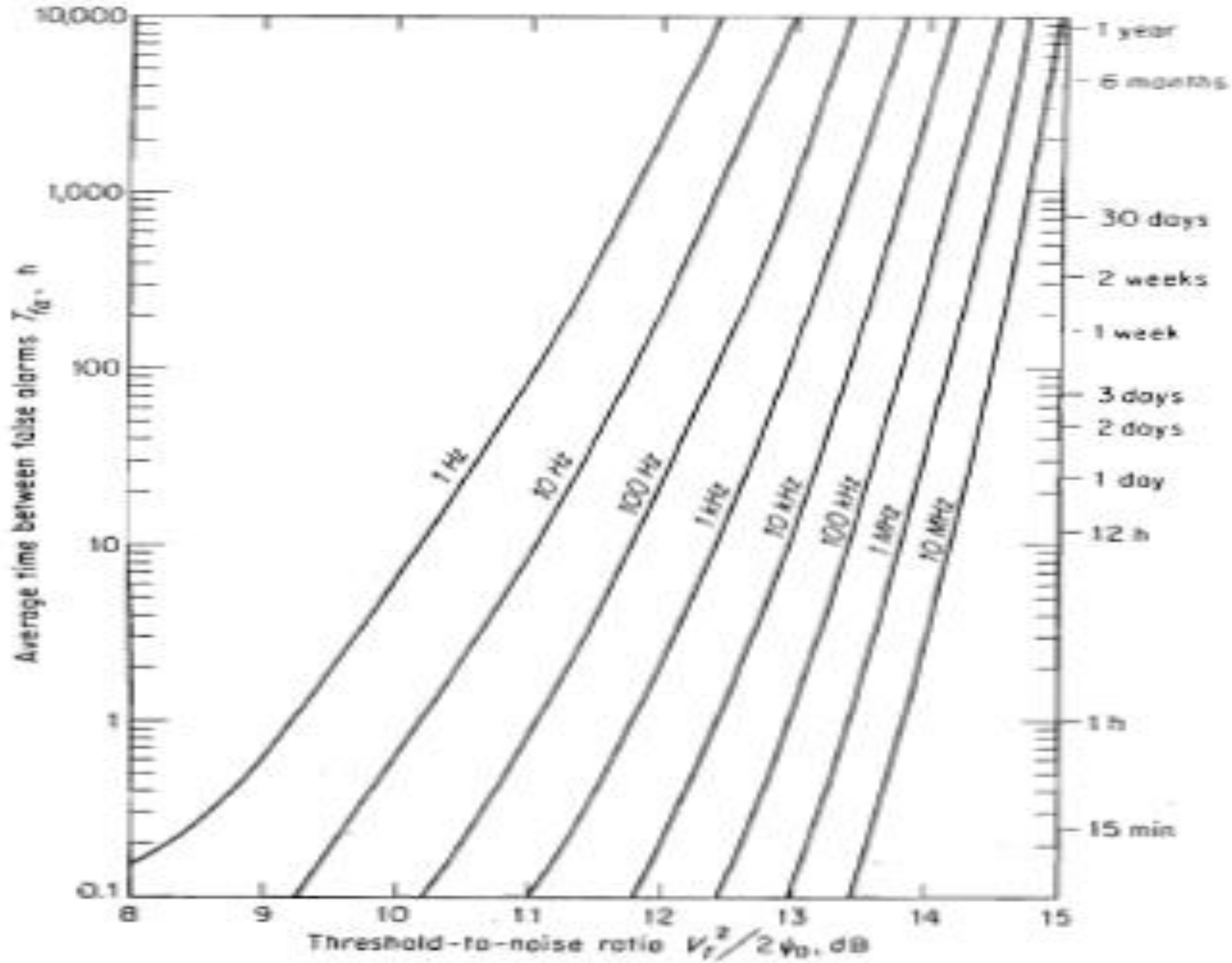
Therefore $T_{fa} = \dfrac{1}{B_{IF}} \exp\left[\dfrac{{V_T}^2}{2\ \varphi_0}\right]$

RELATIONSHIP BETWEEN P_{fa} AND T_{fa}

- i) $P_{fa} = \exp\left[\frac{-{V_T}^2}{2\ \varphi_0}\right]$
- ii) $T_{fa} = \frac{1}{B_{IF}} \exp\left[\frac{{V_T}^2}{2\ \varphi_0}\right]$
- iii) $P_{fa} = \frac{1}{T_{fa}\ B_{IF}}$
- iv) $T_{fa} = \frac{1}{P_{fa}\ B_{IF}}$


1×10^−5
1×10^−6
1×10^−7
1×10^−8
1×10^−9
1×10^−10
1×10^−11
1×10^−12
1×10^−13
1×10^−14
Probability of False Alarm
1×10^−3
0.01
0.1
1
10
100
Average Time Between False Alarms (hours)
B_R
(MHz)
0.1
0.5
1
5
10
50
100

PLOT OF T_{fa} Vs $V_T^{\,2} / 2\,\psi o$



T_{fa} Vs P_{fa} Vs V_T

➢ Example : 1

i) Let Band width = 1 MHz

ii) Average Time between false alarm T_{fa} = 15 Min (900 secs) Find out threshold V_T

- $T_{fa} = \frac{1}{B_{IF}} \exp\left[\frac{{V_T}^2}{2\ \varphi_0}\right]$
- $900 = \frac{1}{1 \times 10^6} \exp\left[\frac{{V_T}^2}{2\ \varphi_0}\right]$
- $9 \times 10^8 = \exp\left[\frac{{V_T}^2}{2\ \varphi_0}\right]$

T_{fa} Vs P_{fa} Vs V_T

- $9 \times 10^8 = \exp\left[\frac{{V_T}^2}{2\ \varphi_0}\right]$
- Taking Logarithms (natural) on both sides
- $20.67 = \left[\frac{{V_T}^2}{2\ \varphi_0}\right]$
- So $V_T = \sqrt{2 \times 20.67\ \varphi_0} = 6.42\sqrt{\varphi_0}$

➢ So $V_T = 6.42 \times \sigma$

➢ Threshold $V_T = 6.42 \times$ RMS value of noise voltage

T_{fa} Vs P_{fa} Vs V_T

- **Example : 2**
- Let B_{IF} = 1 M HZ and 10 log $\frac{{V_T}^2}{2\ \varphi_0}$ = 12.95db
- Find out T_{fa}
- 10 log $\frac{{V_T}^2}{2\ \varphi_0}$ = 12.95db
- log $\frac{{V_T}^2}{2\ \varphi_0} = \frac{12.95}{10}$ = 1.295

T_{fa} Vs P_{fa} Vs V_T

- Taking Antilogarithms with base 10
- $$\frac{V_T^2}{2\ \varphi_0} = 19.724$$
- $$\exp \frac{V_T^2}{2\ \varphi_0} = 368 \times 10^6$$

But $T_{fa} = \frac{1}{B_{IF}} \exp\left[\frac{V_T^2}{2\ \varphi_0}\right] = \frac{1}{10^6} \times 368 \times 10^6 =$

$= 368$ secs $\approx$ 6min

T_{fa} Vs P_{fa} Vs V_T

- **Example : 3**

B_{IF} = 1 M HZ and let 10 log $\frac{-V_T^2}{2\ \varphi_0}$ = 14.72db

Find out T_{fa}

- Calculating the same way as in the above example we have Tfa = 10,000 Hours

- This shows that T_{fa} has changed from 6 minutes to 10,000 Hours with a slight change of threshold by (14.72 – 12.95) = 1.77 dB

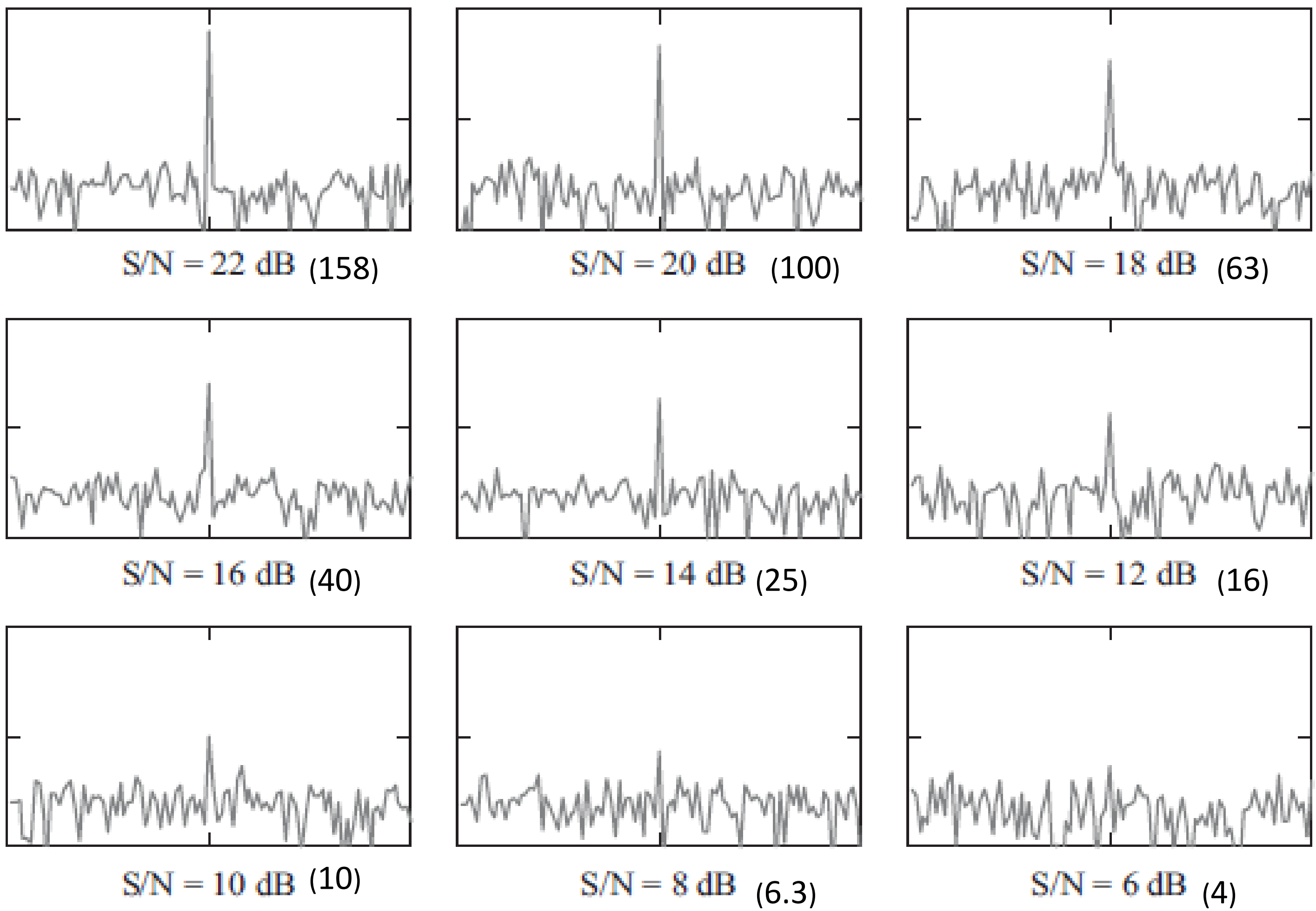
- **So these examples show that False Alarm time is highly sensitive to threshold voltage**

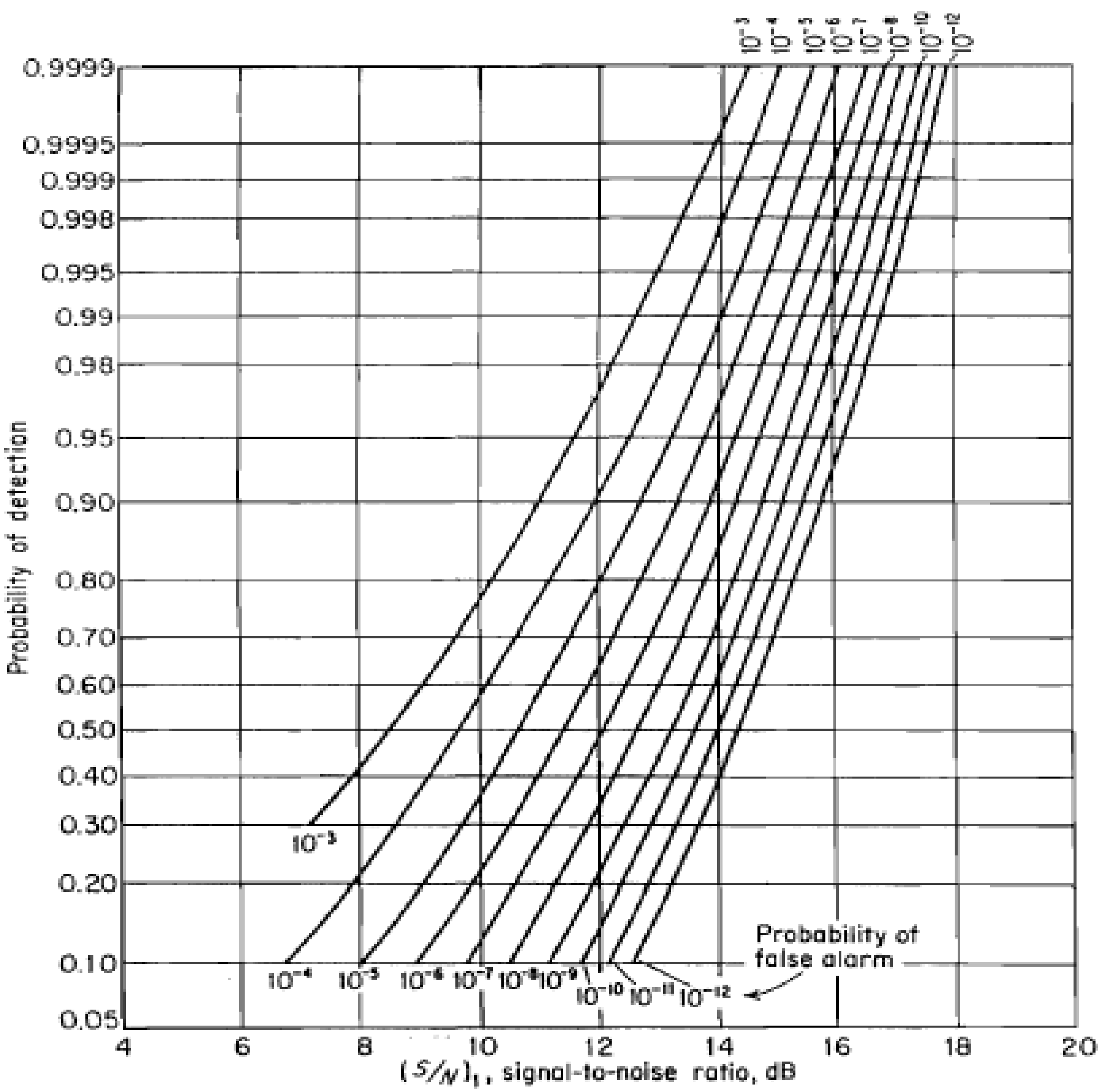
INTEGRATION OF RADAR PULSES

INTEGRATION OF RADAR PULSES

- (Jntuh) Discuss the effect of integration of radar pulses
- Radar range equation is given as
- $R_{max}{}^{4} = \dfrac{P_T \; G \; A_e \; \sigma}{(4\pi)^2 \; K \; T_0 \; B \; F_n \; (S/N)_1}$

Target Detection





Probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm.

INTEGRATION OF RADAR PULSES

- **Necessity for Integration :**

Radar range equation is given as

$$R_{max}^{\;4} = \frac{P_T \; G \; A_e \; \sigma}{(4 \; \pi)^2 \; K \; T_0 \; B \; F_n \; (S/N)_1}$$

where $(S/N)_1$ is the Signal to Noise ratio for a single pulse

- One of the ways to increase R_{max} is by decreasing $(S/N)_1$
- This is accomplished by **Pulse Integration**

INTEGRATION OF RADAR PULSES (CONTD….)

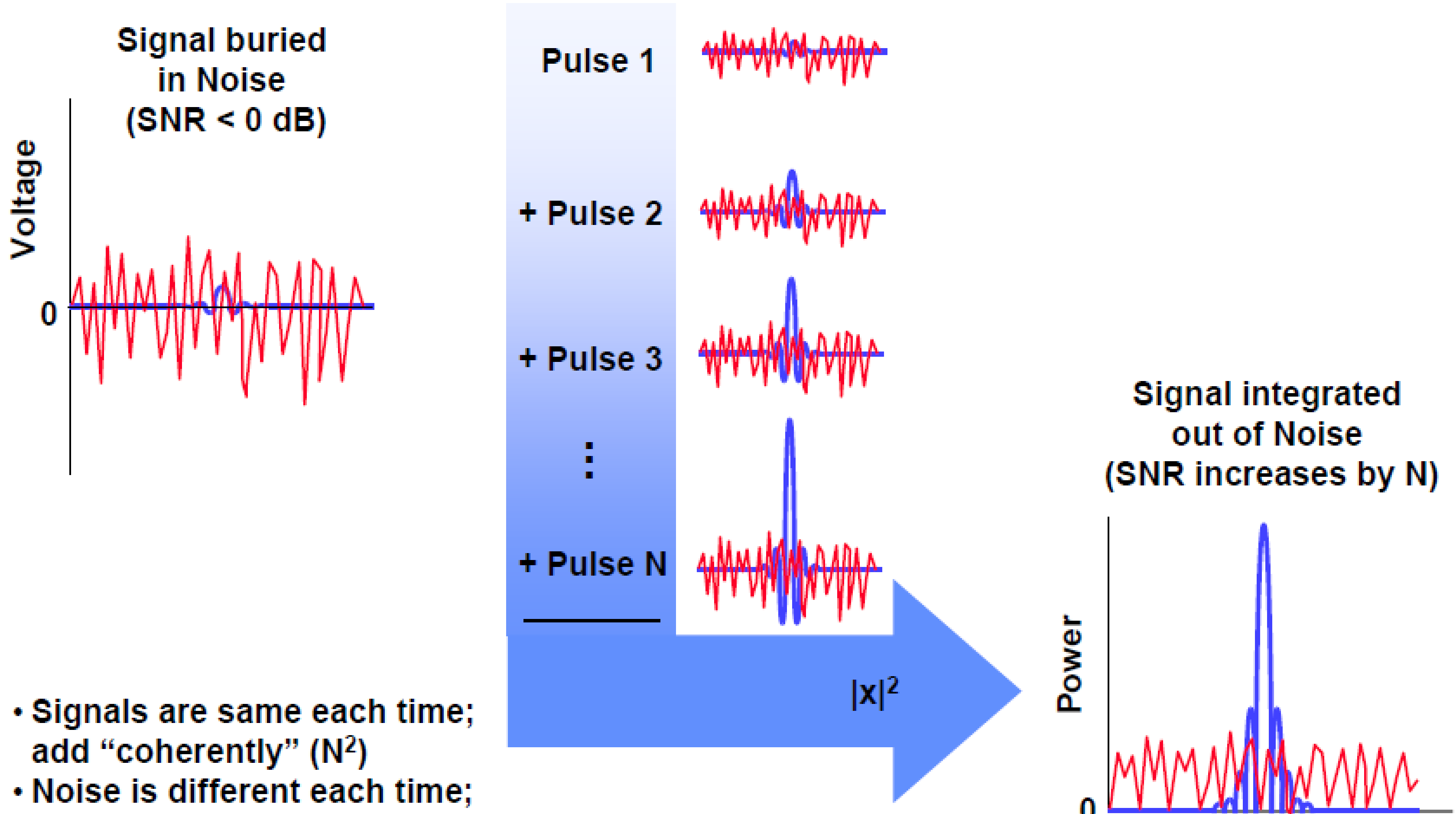
- In the Radar equation $(S/N)_1$ need to be replaced by $(S/N)_n$ where $(S/N)_1$ is the signal to noise ratio for a single pulse and $(S/N)_n$ is the Signal to noise ratio for **'n'** number of pulses

- $$R_{max}{}^4 = \frac{P_T \; G \; A_e \; \sigma}{(4\;\pi)^2 \; K \; T_0 \; B \; F_n \; (S/N)_n}$$

- $$R_{max}{}^4 = \frac{P_T \; G \; A_e \; \sigma}{(4\;\pi)^2 \; K \; T_0 \; B \; F_n \; \frac{(S/N)_1}{n}}$$

- $$R_{max}{}^4 = \frac{P_T \; G \; A_e \; \sigma \; n}{(4\;\pi)^2 \; K \; T_0 \; B \; F_n \; (S/N)_1}$$

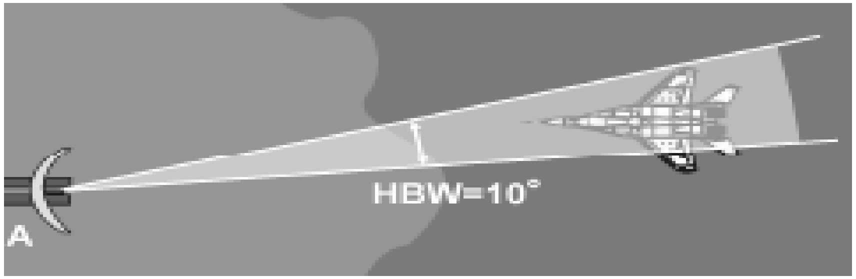
Coherent Integration



- Signals are same each time; add "coherently" (N^2)
- Noise is different each time; doesn't add coherently (N)

INTEGRATION OF RADAR PULSES (CONTD…)

- **Number of Pulses available for Integration in a Search Radar:**



$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

- Time for which beam illuminates target = $\frac{\theta_B}{\dot{\theta}_S}$

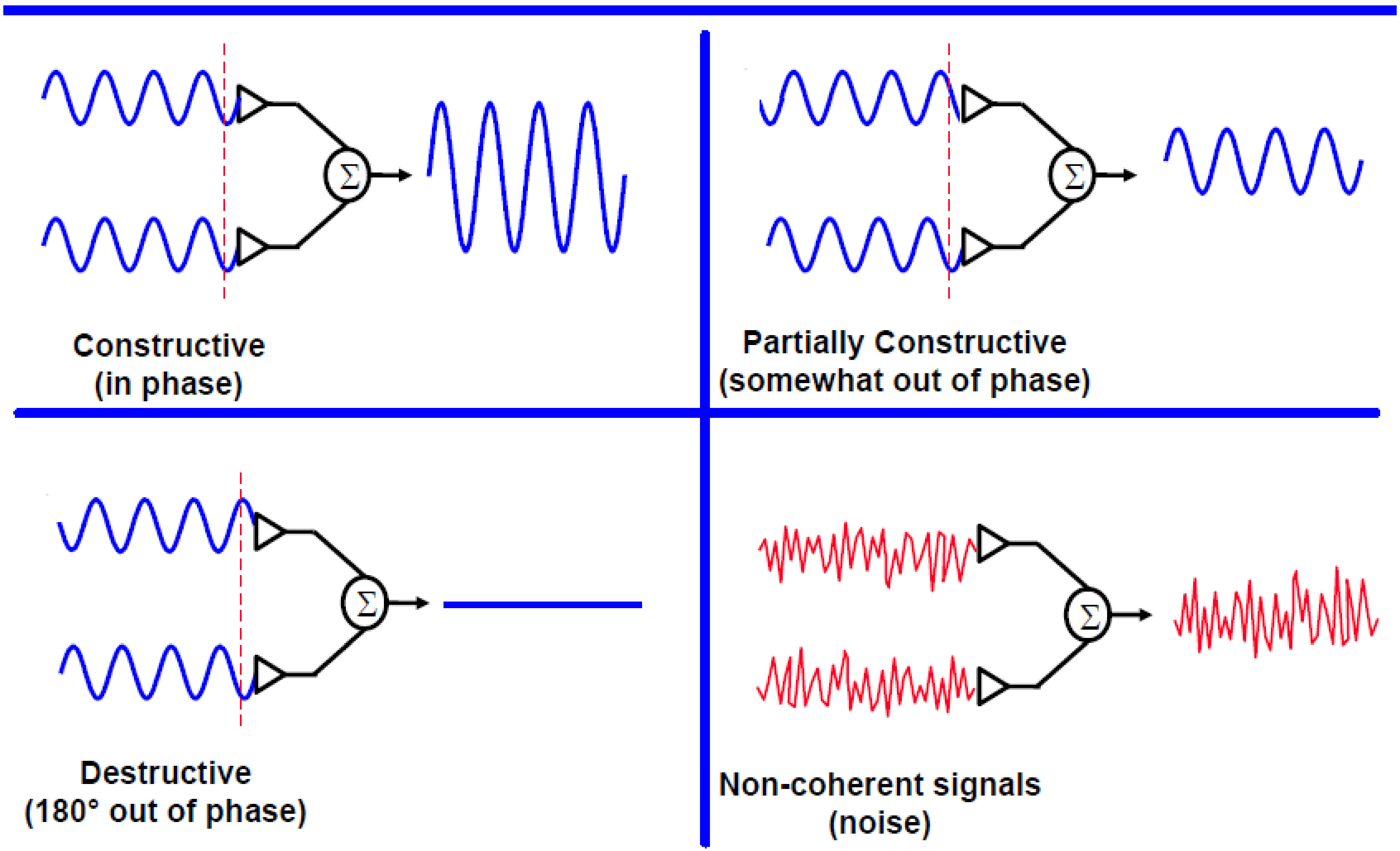
$$\frac{\text{Beam width}}{\text{Angular speed}}$$

INTEGRATION OF RADAR PULSES (CONTD…)

- Number of Pulses returned from a target "n"
- n = $\frac{\theta_B}{\dot{\theta}_S} \times f_P$ where f_p = PRF =No. of pulses/sec
- $\dot{\theta}_S = \omega_S \times \frac{360}{60} = 6\ \omega_S$
- Angular Speed =

  where ω_s = Revolutions of antenna per minute
- So $n = \frac{\theta_B}{6\ \omega_S} \times f_P$

Constructive vs. Destructive Addition



Constructive
(in phase)

Partially Constructive
(somewhat out of phase)

Destructive
(180° out of phase)

Non-coherent signals
(noise)

INTEGRATION OF RADAR PULSES (CONTD….)

- **Pulse Integration:**

  Process of summing vectorially all Radar echoes from a target is called Pulse Integration

- **Methods for integration:**

  i) Take advantage of persistence of Phospher of CRT display combined with integrating properties of eye

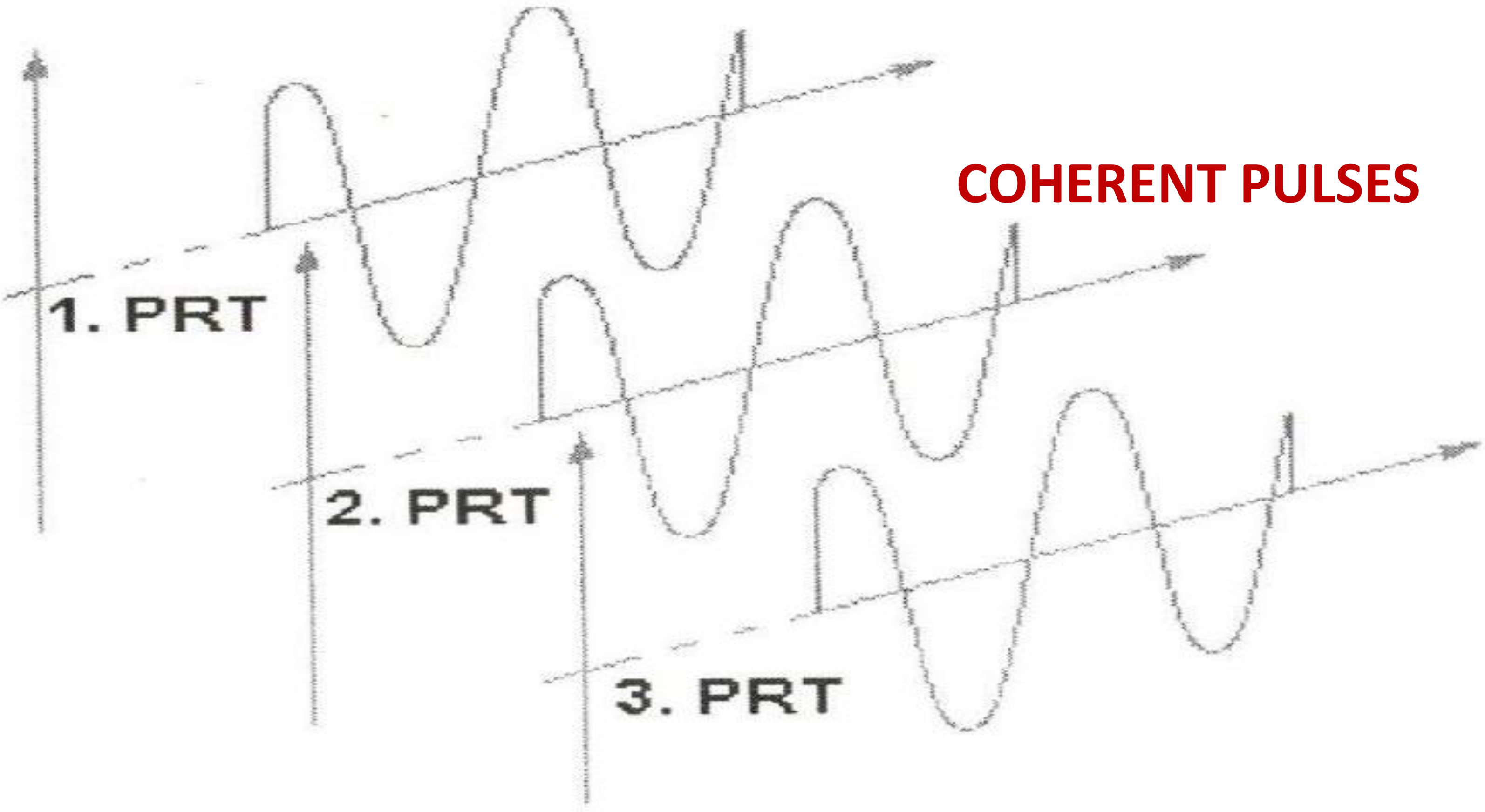
  ii) Analog or Digital method of integration

- **Types of Integration:**
- i) Coherent or Pre-detection integration
- ii) Non coherent or Post integration

INTEGRATION OF RADAR PULSES (CONTD…)

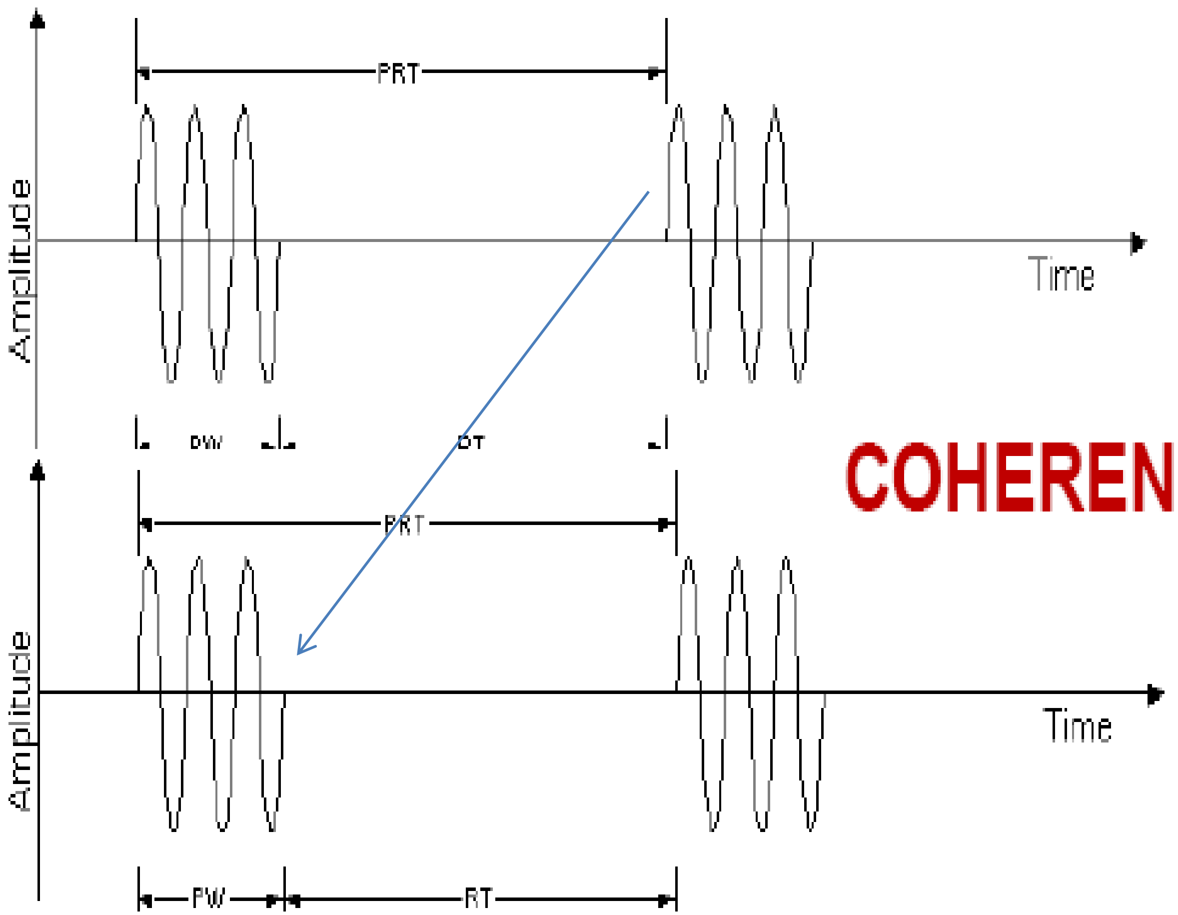
- (i) **Coherent Integration:**
- Integration is accomplished in IF (before second detector)
- The phase of echo signal is preserved for summing up with the next pulse.
- More efficient than noncoherent integration.
- If ‘n’ pulses were integrated the resultant S/N ratio would be exactly ‘n’ times the S/N ratio of single pulse.
- The integration is difficult to implement and consists of a narrow band comb IF filter.
- The phase of IF carrier oscillation be maintained coherent over a time corresponding to the time on target.

INTEGRATION OF RADAR PULSES (CONTD…)



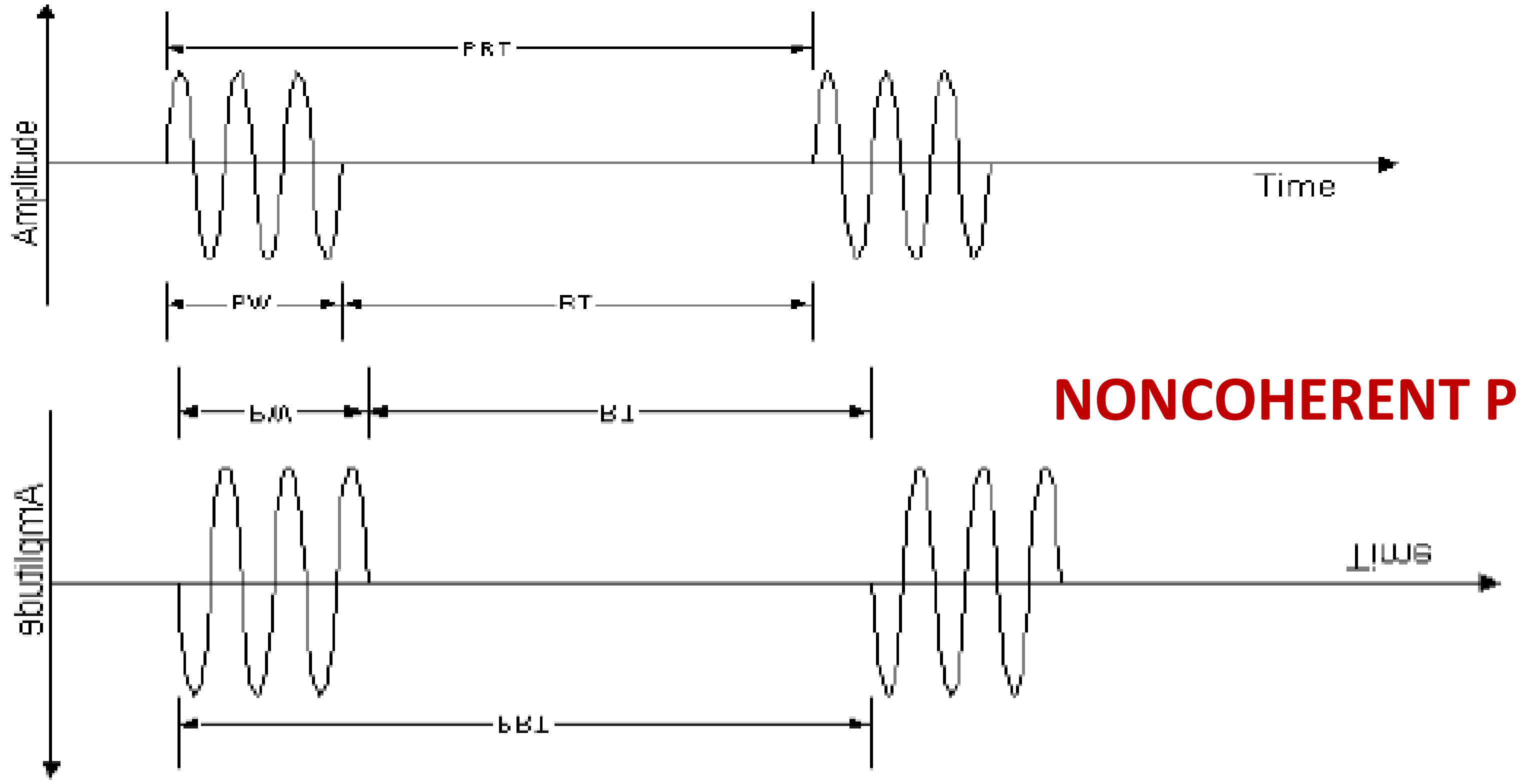
COHERENT PULSES

INTEGRATION OF RADAR PULSES (CONTD…)



COHERENT PULSES

INTEGRATION OF RADAR PULSES (CONTD…)



NONCOHERENT PULSES

INTEGRATION OF RADAR PULSES (CONTD…)

ii) Non coherent (Post) Integration:

- Integration is accomplished in video (after the second detection).
- Phase information is destroyed by the second detector and phase information about echo (RF) pulse is not available.
- The pulses that are summed are video pulses.
- If n pulses are integrated the resultant S/N is less than n times S/N of single pulse.
- The loss in efficiency is due to the nonlinear action of the detector.
- The integration is easier to implement and consists of low pass filter in the video portion of the Receiver.

INTEGRATION OF RADAR PULSES (CONTD…)

- **Efficiency of Integration $E_i(n)$**

$$E_i(n) = \frac{(S/N)_1}{n\ (S/N)_n}$$

where n = number of pulses integrated

$(S/N)_1$ = S/N ratio of single pulse required to produce given probability of detection.

$(S/N)_n$ = S/N ratio per pulse required to produce same probability when 'n' pulses are integrated.

- **Integration Improvement factor : $I_i(n)$**

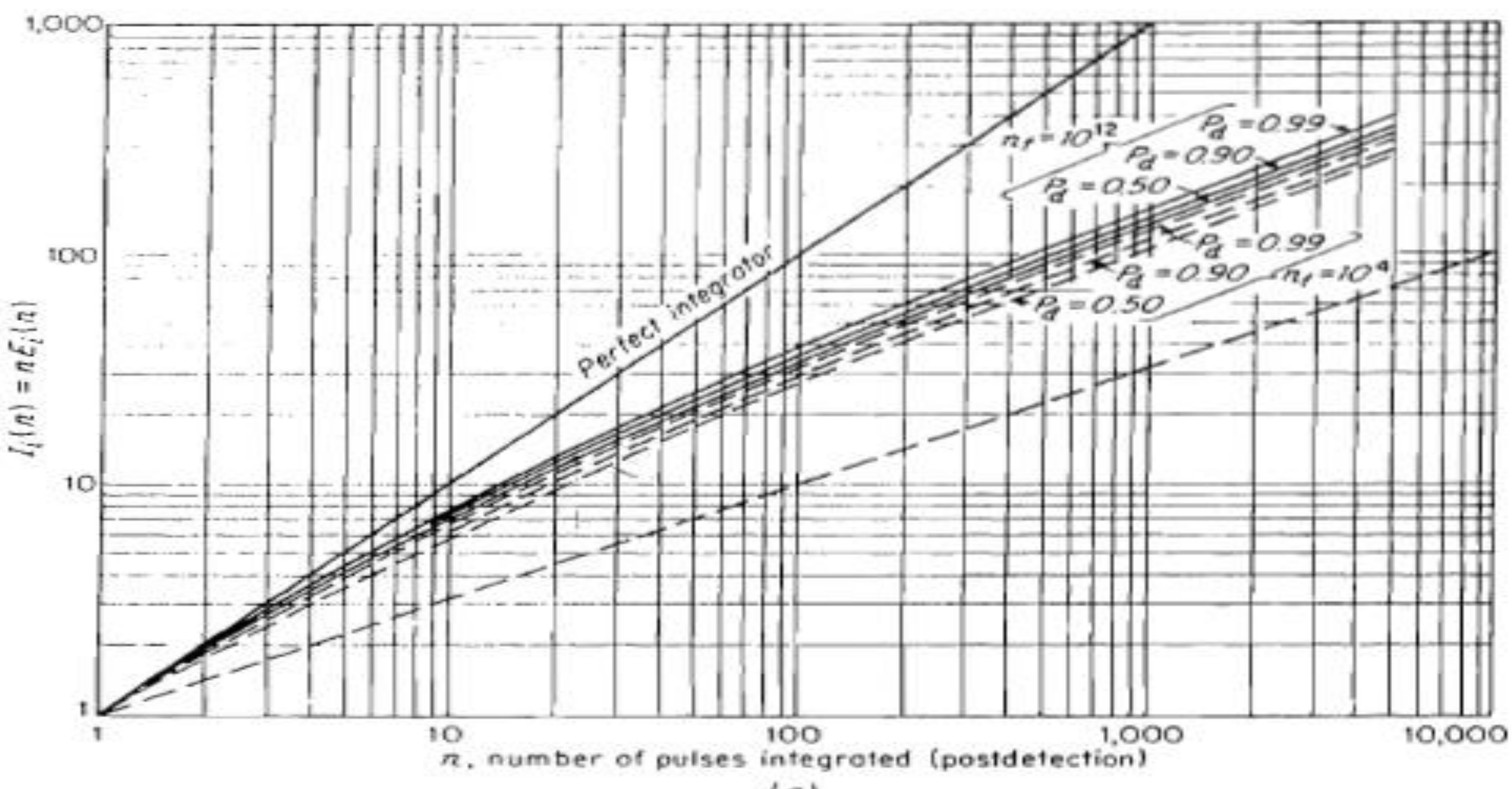
  $I_i(n)$ for pre integration = n

  $I_i(n)$ for post detection = $n\,E_i(n)$

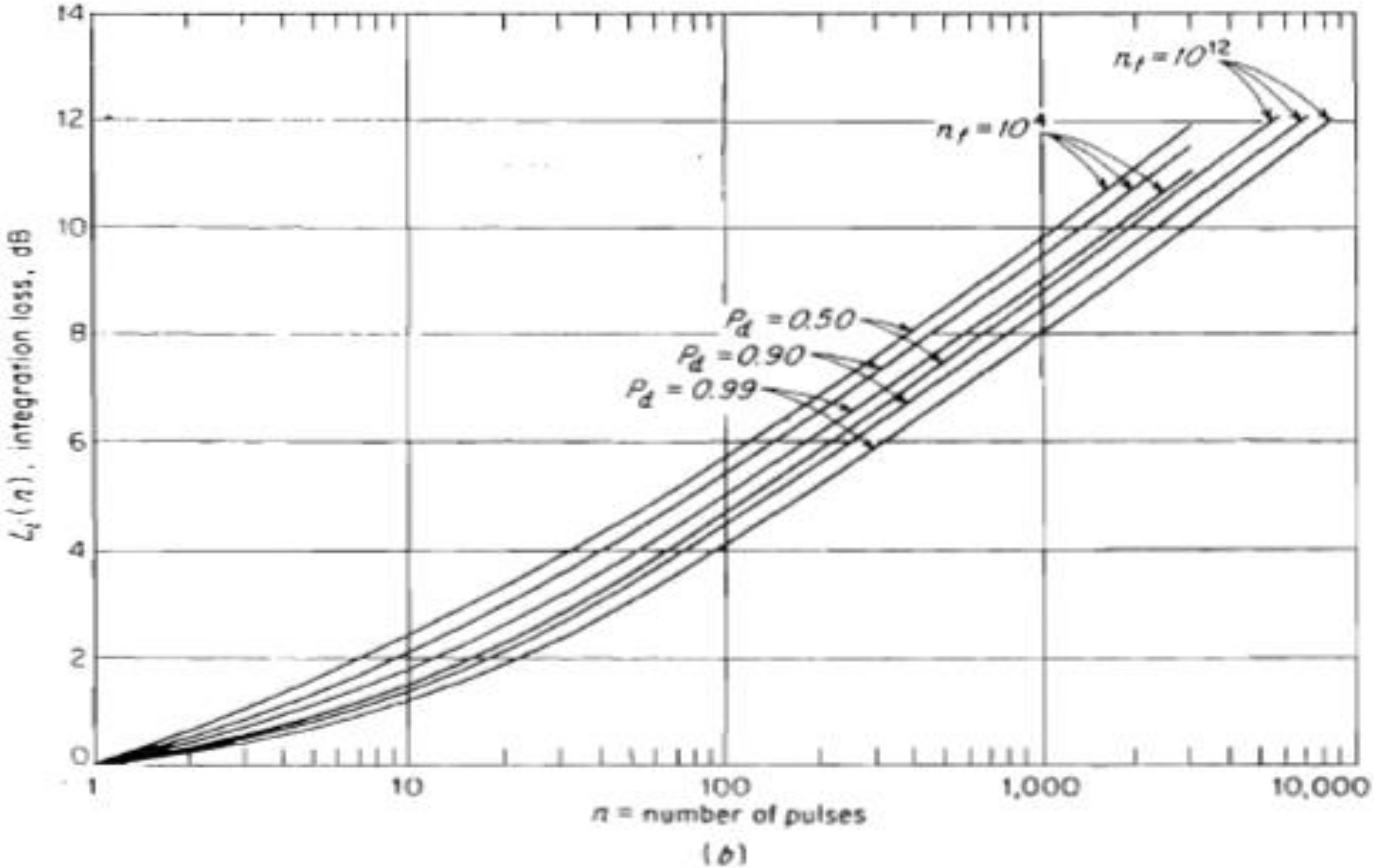
INTEGRATION OF RADAR PULSES (CONTD….)

- Integration Loss : L_i (n)

$$L_i(n) = 10\,\mathrm{Log}_{10}\left[\frac{1}{E_{i(n)}}\right]$$



INTEGRATION OF RADAR PULSES (CONTD…)



INTEGRATION OF RADAR PULSES (CONTD…)

- The I_i (n) or L_i (n) are not sensitive functions of either probability of detection or probability of false alarm.

INTEGRATION OF RADAR PULSES (CONTD…)

- How to obtain signal-to-Noise Ratio per pulse $(S/N)_n$ for specified P_d and $P_{fa,}$ when 'n' pulses are integrated.
- **Steps:**

1. For specified T_{fa}, B and 'n' compute P_{fa}

False Alarm Probability $P_{fa} = \dfrac{n}{T_{fa}\ B} = \dfrac{n}{T_{fa}\ f_p\ \eta}$

When f_p = Pulse Repetition frequency

η = number of pulse intervals per radar sweep (no. of pulse repetition periods)

INTEGRATION OF RADAR PULSES (CONTD…)

2. For specified θ, Pd (Probability of detection) and computed P_{fa} (from step 1 above)

   Find the signal-to-noise ration $(S/N)_1$ for single pulse detection from the graph.

3. False alarm number $n_f = \frac{n}{P_{fa}} = T_{fa}\,B$

   Find the Integration improvement factor nEi(n) from the graph

4. $(S/N)_n = \frac{(S/N)_1}{n\ Ei(n)}$

INTEGRATION OF RADAR PULSES (CONTD…)

- The new Radar Equation because of Integration

$$R_{max}{}^{4} = \frac{P_T \; G \; A_e \; \sigma \; n \; E_i\,(n)}{(4\,\pi)^2 \; K \; T_0 \; B_n \; F_n \; (S/N)_1}$$

INTEGRATION OF RADAR PULSES (CONTD…)

(i) Exponential Weighting:

- Practical Integrators do not sum the echo pulses with equal weight.
- $V = \sum V_i \exp[-(i-1)\gamma]$

  where V_i = Voltage amplitude of 'i' th pulse

  $\exp(-\gamma)$ = attenuation factor per pulse

- Pulse 1, the last pulse received is given a weight of unity '1'
- Pulse 2, is attenuated by a factor $e^{-\gamma}$
- Pulse 3, is attenuated by $e^{-2\gamma}$
- Pulse n, is attenuated by $e^{-(n-1)\gamma}$
- Exponential weighting results in less efficient integration than uniform integration.

INTEGRATION OF RADAR PULSES (CONTD…)

- **(ii) Dumped Integrator:**

- Used with step-scan Radar
- In step-scan Radar, antenna remains stationary until 'n' pulses are received, after which it is stepped to next position.

- Example:

(i) Electrostatic storage tube that is erased after reading

(ii) Capacitor that is discharged after read-out

INTEGRATION OF RADAR PULSES (CONTD…)

- Efficiency of integrators

If ρ =

$$\frac{Average\ Signal\text{-}to\text{-}noise\ ratio\ for\ the\ exponential\ integrator}{Average\ Signal\text{-}to\text{-}noise\ ratio\ for\ the\ uniform\ integrator}$$

- Efficiency ρ for Dumped Integrator

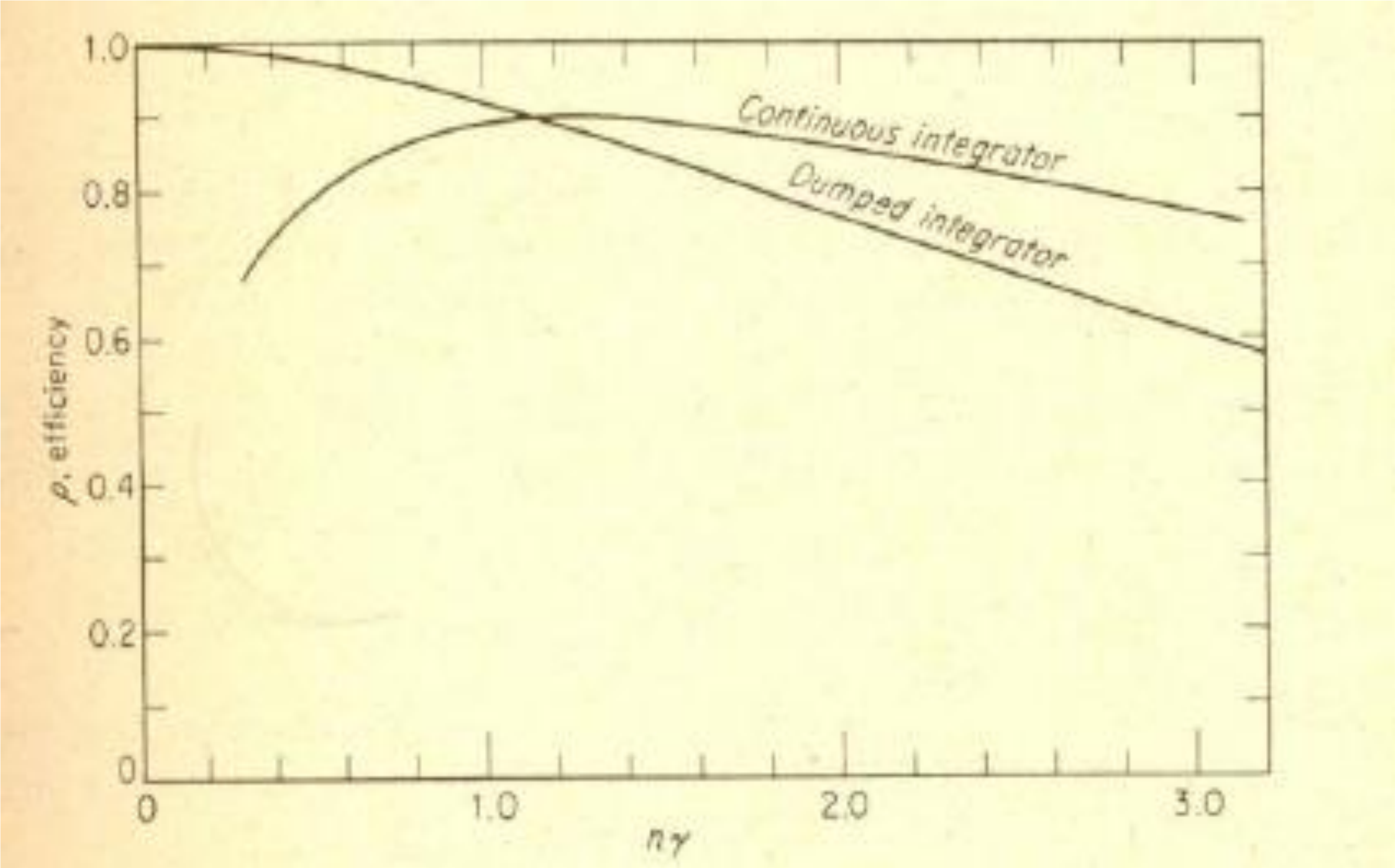
$$\rho = \frac{\tanh(\frac{n\gamma}{2})}{n\tanh(\frac{\gamma}{2})}$$

where n = number of pulses

γ = attenuation factor per pulse (usually small)

INTEGRATION OF RADAR PULSES (CONTD…)

- Graph shows the ρ efficiency of continuous and dumped integrator



INTEGRATION OF RADAR PULSES (CONTD…)

- **Efficiency of Integrators (contd..)**

❖ Efficiency for continuous exponential weighting

$$\text{integrator } \rho = \frac{[1 - \exp(-n\,\gamma)]^2}{n\ \tan(\frac{\gamma}{2})}$$

➢ Maximum efficiency for continuous integrator occurs for (n γ) = 1.257

➢ Maximum efficiency for dumped integrator occurs for γ = 0

CONTINUED IN RADAR 1 F

RADAR SYSTEMS
(EC 812 PE)
(ELECTIVE V)
UNIT – 1 F
B.TECH IV YEAR II SEMESTER

BY

Prof.G.KUMARASWAMY RAO

(Former Director DLRL Ministry of Defence)

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Acknowledgements

The contents , figures , graphs etc., are taken from the following Text book & others

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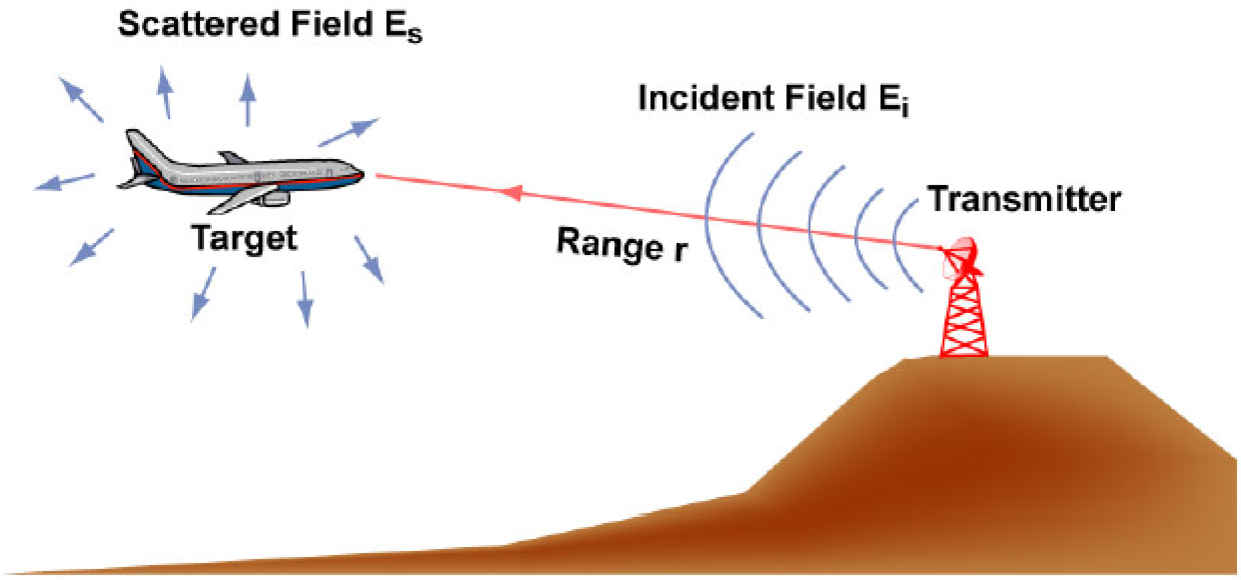
RADAR CROSS SECTION (RCS)

RADAR CROSS SECTION (RCS)

- RCS is the property of a scattering object.
- RCS represent the magnitude of the signal returned to the radar (echo).
- RCS (σ) is included in the Radar equation.

- $$R^4_{max} = \frac{P_T \; G \; A_e \; \sigma \; n \; E_I(n)}{(4\pi)^2 \; K \; T_0 \; B_n \; F_n \; (S/N)_1}$$

Definition of Radar Cross Section (RCS or σ)



$$RCS = \lim_{r \to \infty} 4\pi r^2 \frac{|E_s|^2}{|E_i|^2} \quad \text{(Unit: Area)}$$

Radar Cross Section is the area intercepting that amount of power which, if radiated isotropically, produces the same received power in the radar.

RADAR CROSS SECTION (RCS) (CONTD…)

- **RCS Definitions:** (i) RCS of a target is the area intercepting that amount of power which, when scattered equally in all directions, produces an echo at Radar equal to that from target.

  (ii) RCS is a fictional area that intercepts a part of power incident at the target, which, if scattered uniformly in all directions, produces an echo power at the radar equal to that produced at the radar by the real target.

Real targets do not scatter the energy uniformly in all directions.

RADAR CROSS SECTION (RCS) (CONTD…)

$$\sigma = \frac{\text{Power reflected toward source/solid angle}}{\text{Incident power}/4\pi}$$

$$= \lim_{R \to \infty} 4 \pi R^2 \left[\frac{E_r}{E_I}\right]^2$$

Where E_r = Reflected electric field strength of echo at Radar

E_i = Reflected electric field strength incident on target

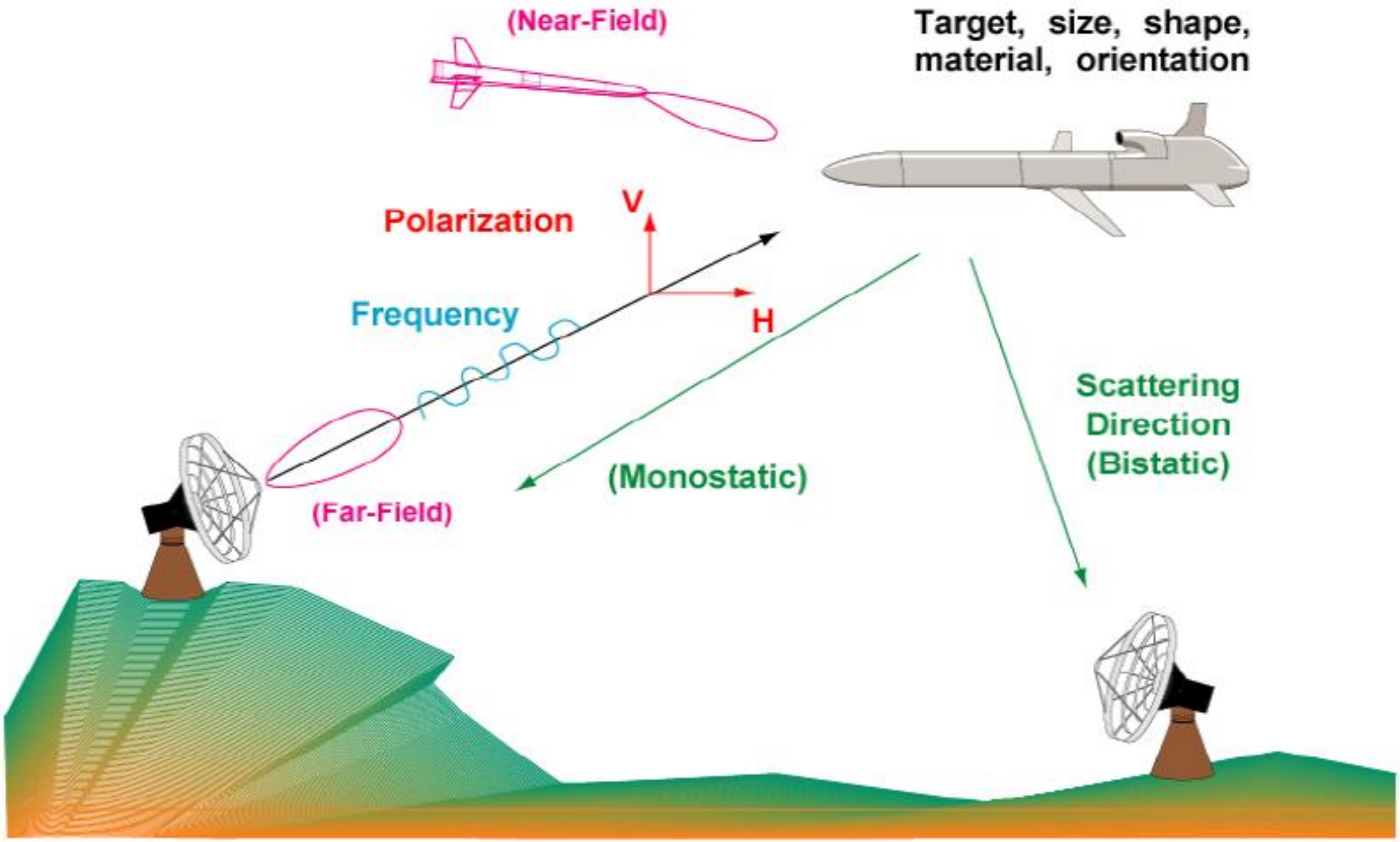
R = Range

- It is assumed the target is far enough ($R \to \infty$)

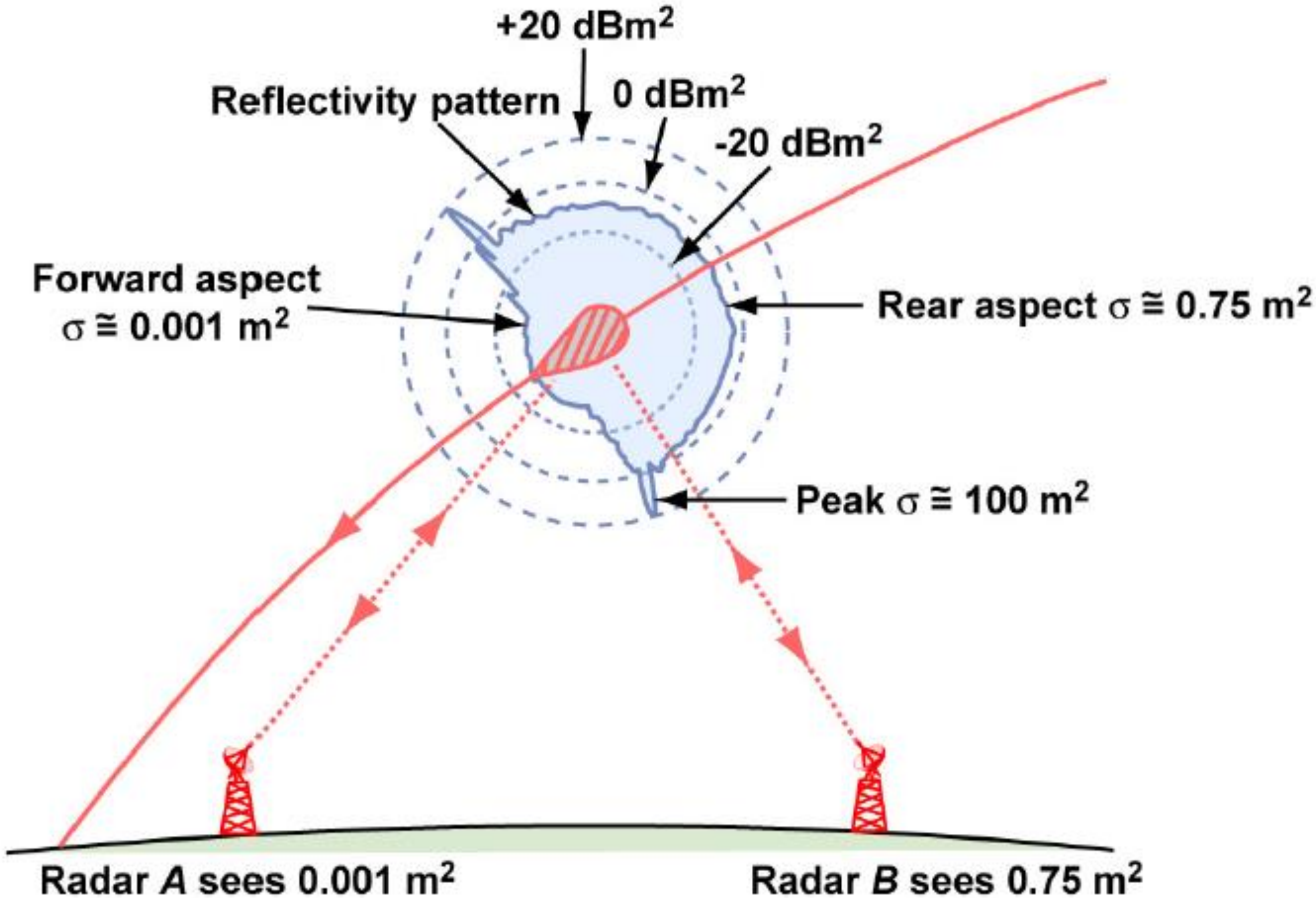
so that incident wave can be considered to be planar.

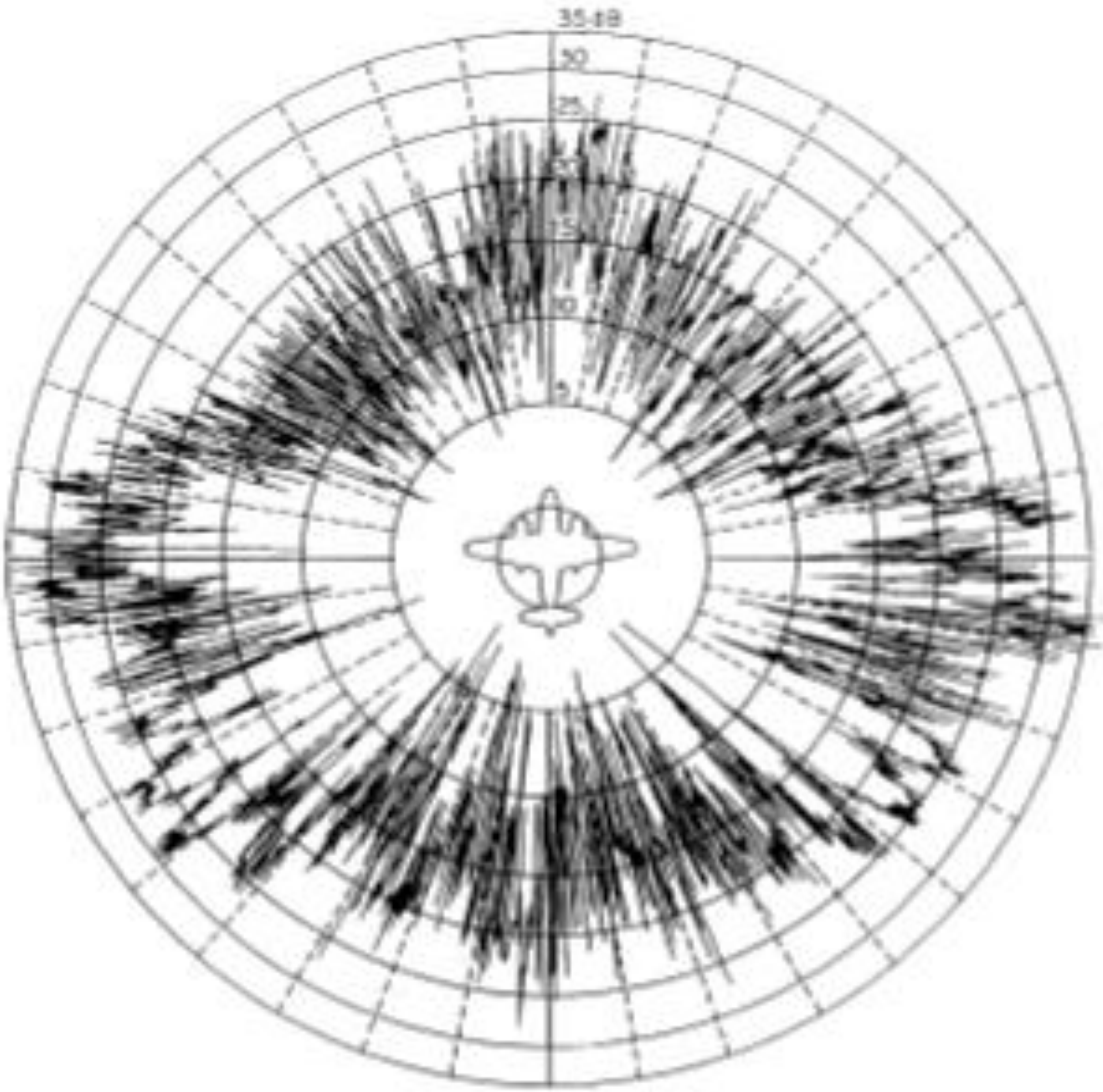
$\longrightarrow$

Factors Determining RCS



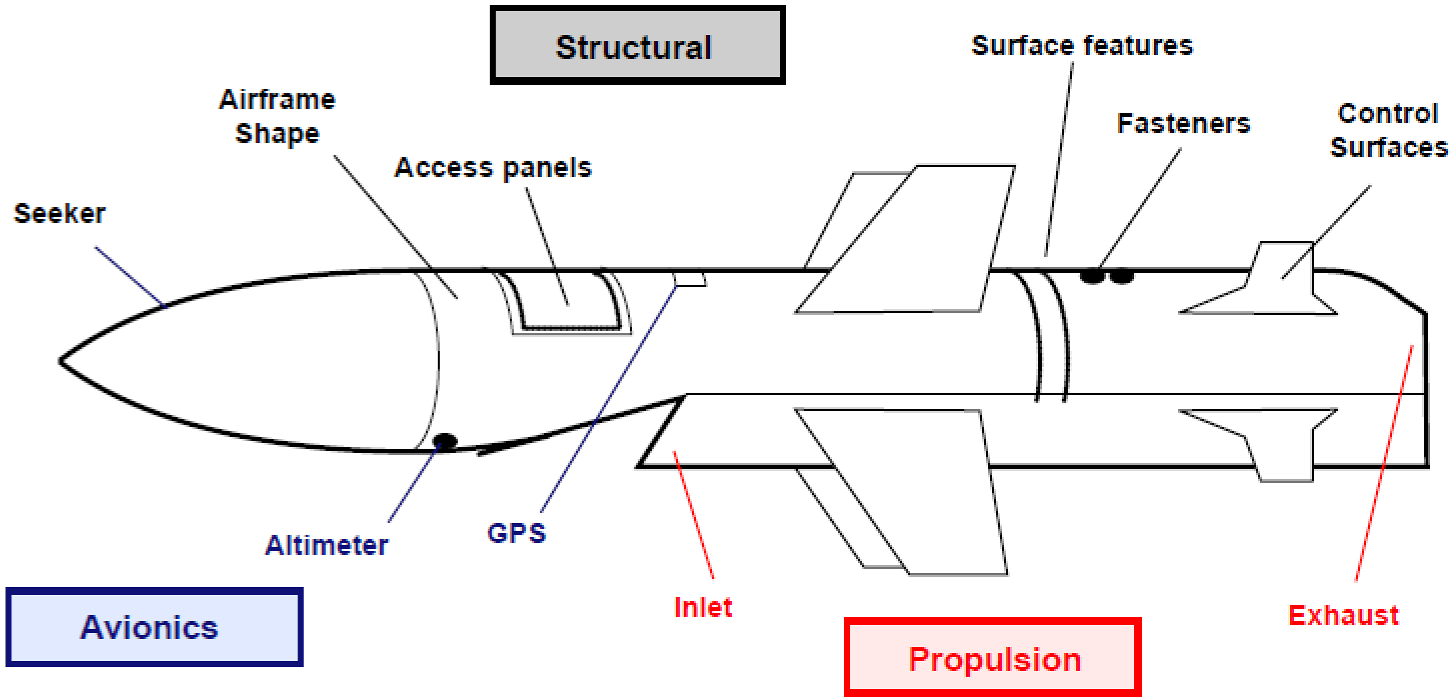
Radar Cross Section of Typical RV





RCS (σ) OF AN AIRCRAFT

Components of Target RCS



- Three types of RCS contributors:
  - Structural (body shape, control surfaces, etc.)
  - Propulsion (inlets, exhaust, etc.)
  - Avionics (seeker, GPS, altimeter, etc.)

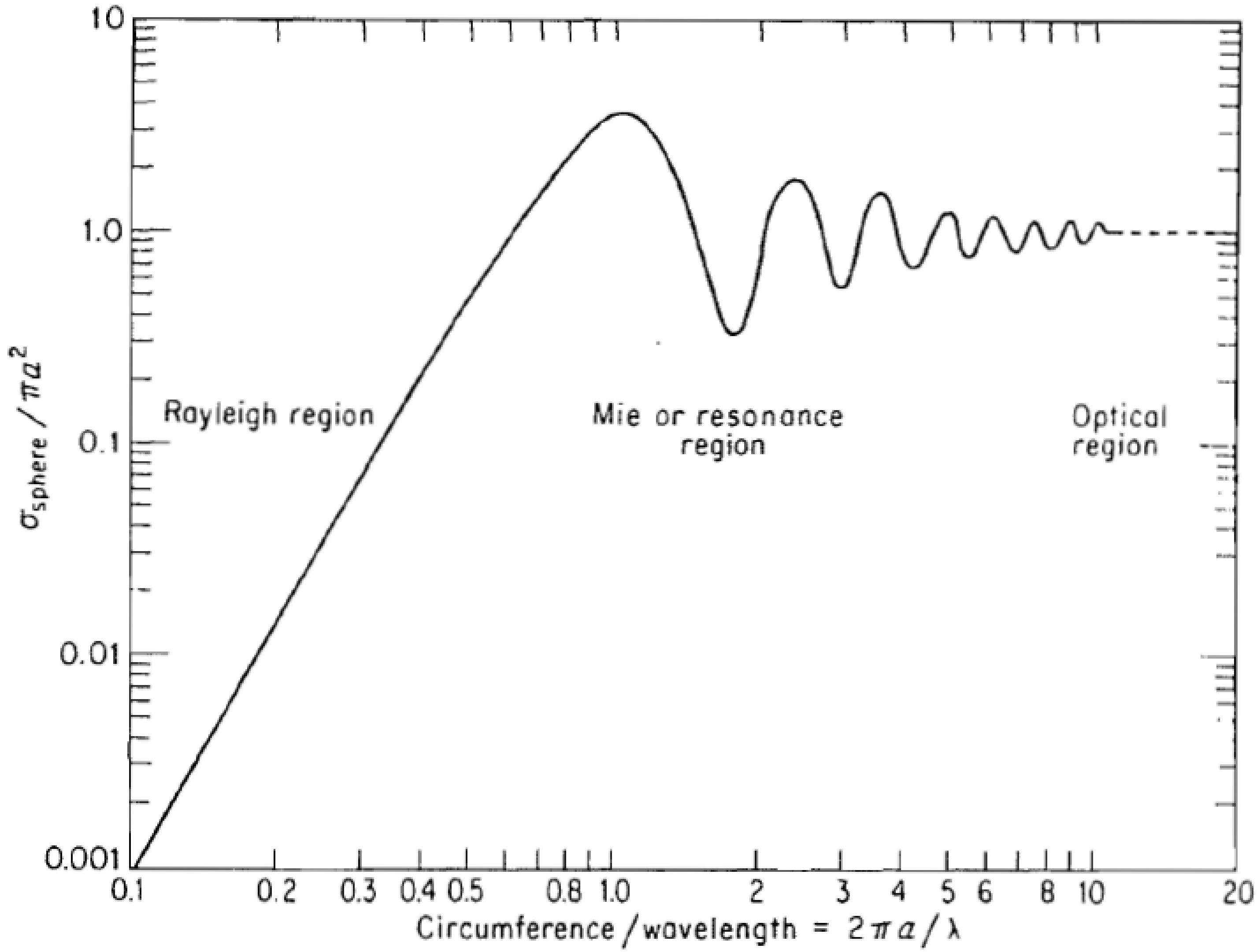
Target on support

- Foam column mounting
  - Dielectric properties of styrofoam close to those of free space

- Metal pylon mounting
  - Metal pylon shaped to reduce radar reflections
  - Background subtraction can be used

RADAR CROSS SECTION (RCS) (CONTD…)

- Maxwell's equations with proper boundary conditions are applied to obtain RCS.
- RCS for simple shapes are obtained by Maxwell equations valid for large range of frequencies.
- For complex shapes solutions are not easy to obtain
- Simple shapes are
  (i) Sphere (ii) Long Thin Rod (iii) Cone sphere



Radar cross section of the sphere. a = radius; λ = wavelength.

RADAR CROSS SECTION (RCS) (CONTD…)

- RCS of sphere is characterized into 3 (Three) regions.
  (i) Rayleigh Region (ii) MIE or Resonance Region
  (iii) Optical Region

- **Rayleigh Region:**

- In 1870 Lord Rayleigh conducted experiments on scattering of light by Microscopic particle. Same is applicable to radar.
- In this region size of sphere is small compared to λ i.e. $\frac{2\pi a}{\lambda} << 1$

- Raindrops and other meteorological particles falls within this region.

RADAR CROSS SECTION (RCS) (CONTD…)

- RCS of objects in this region varies as λ^{-4} , rain and clouds are invisible to radars which operate at long wavelengths (low frequencies)
- (However Rain Drop echoes are required for meteorological radars. So higher radar frequencies are used for meteorological radars).

RADAR CROSS SECTION (RCS) (CONTD…)

➢ MIE or Resonance Region:

- The RCS is oscillatory in this region.
- Maximum value is 5.6 dB (3.63) greater than optical value (πa^2) and value of first null is 5.5 dB (0.275) below optical value.

➢ Optical Region

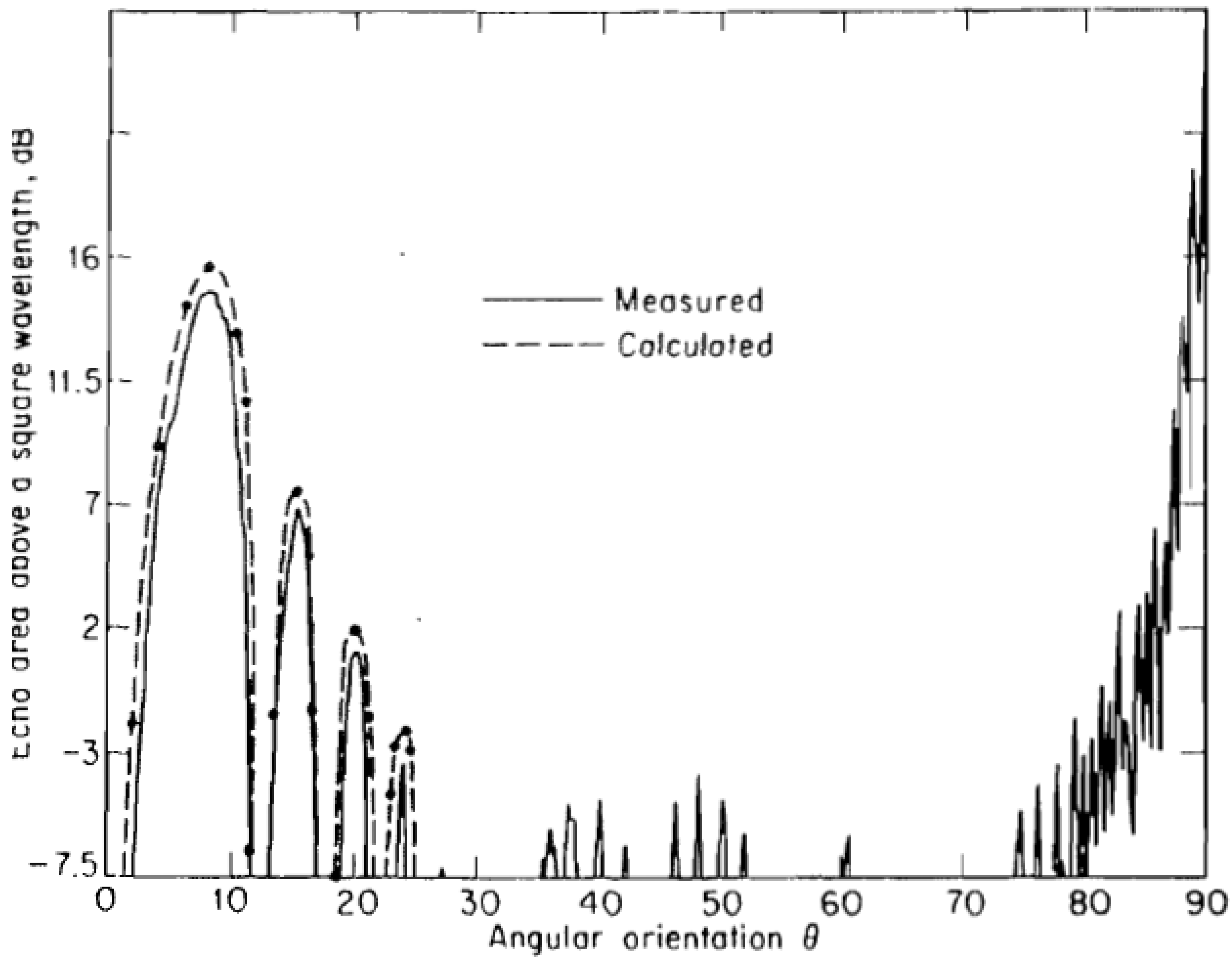
- Dimensions of sphere are large compared to λ

  ie.,($\frac{2 \pi a}{\lambda}$ >> 1)

- In this region the RCS approaches optical RCS

  i.e. πa^2

- RCS of sphere is same from all aspect viewing angles unlike other objects.



Backscatter cross section of a long thin rod. (*From Peters,*[26] *IRE Trans.*)

Radar cross section of the sphere. a = radius; λ = wavelength.

RADAR CROSS SECTION (RCS) (CONTD…)

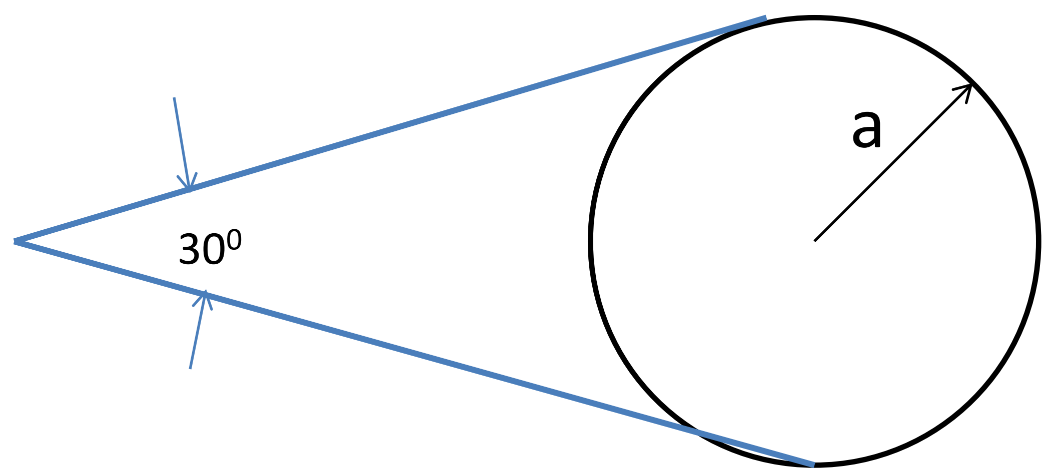
- **RCS Long Thin Rod (Contd…)**

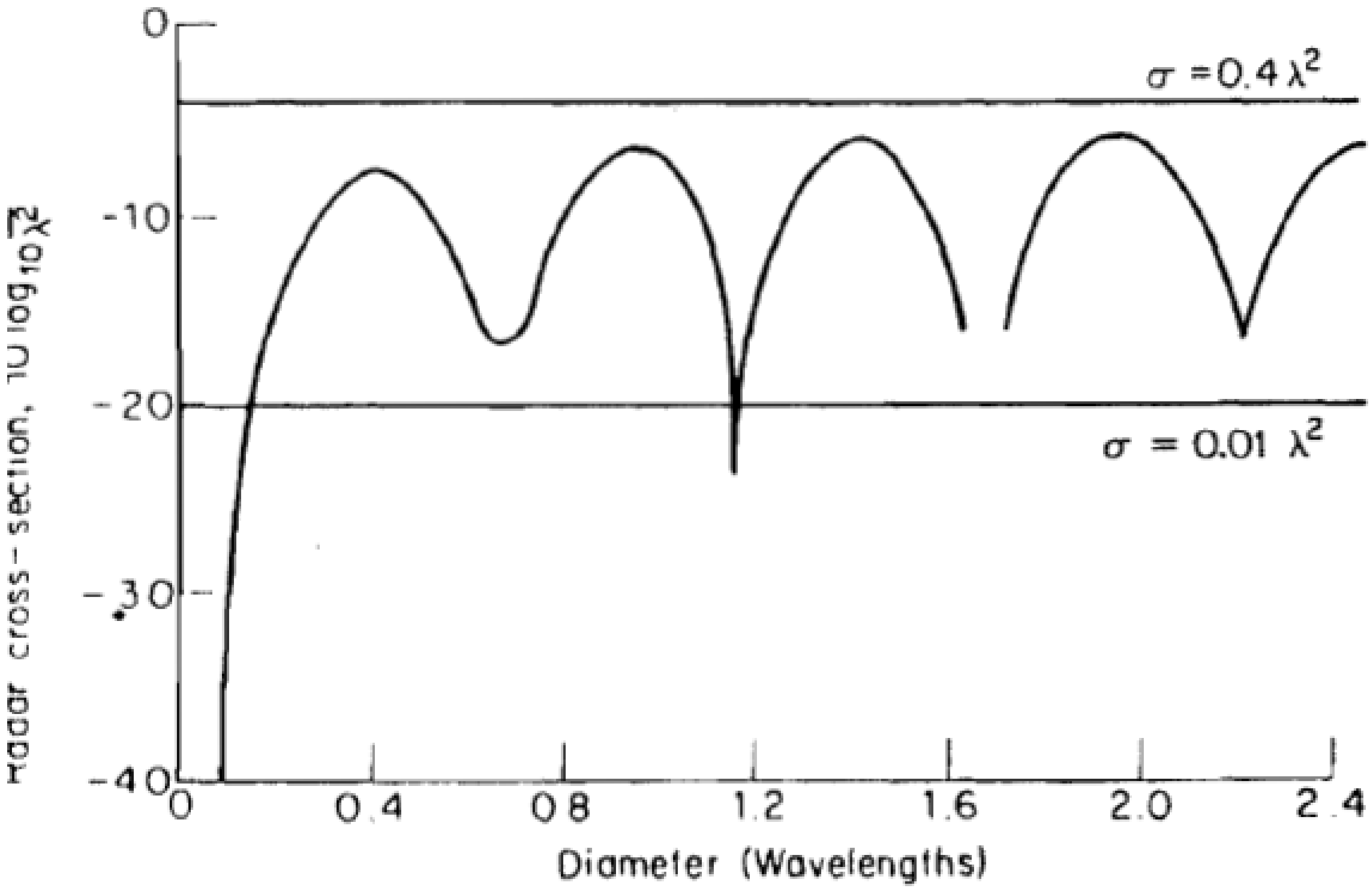
- If the rod is made of steel instead of silver the first maximum would be 5 dB below that shown in the graph
- Viewed end on $\Theta = 0^0$, RCS is small (physical area is small)
- Viewed broad scale $\Theta = 90^0$ RCS is large.
- However as Θ increases, RCS levels off and then increases.

RADAR CROSS SECTION (RCS) (CONTD…)

➢ **RCS of cone sphere:**

- Cone base is capped with sphere
- First derivatives of cone and sphere contours are same at the joining between two.





Radar cross section of a cone sphere with 15° half angle

RADAR CROSS SECTION (RCS) (CONTD…)

➢ RCS of Cone Sphere (Contd…)

- RCS of cone-sphere is low from nose on to near normal incidence on the side of the cone.
- RCS does not depend significantly on cone angle or volume.
- From the rear, the RCS is that of sphere much larger than viewed from front.
- RCS is large when viewed from an angle perpendicular to its surface ($\Theta = 90 - \alpha$ where α = cone half angle)
- Nose on RCS is 0.4 λ^2 maximum and .01 λ^2 minimum

RADAR CROSS SECTION (RCS) (CONTD…)

➢ Reducing RCS of Cone Sphere:

(i)Tip of cone must be sharp not rounded.

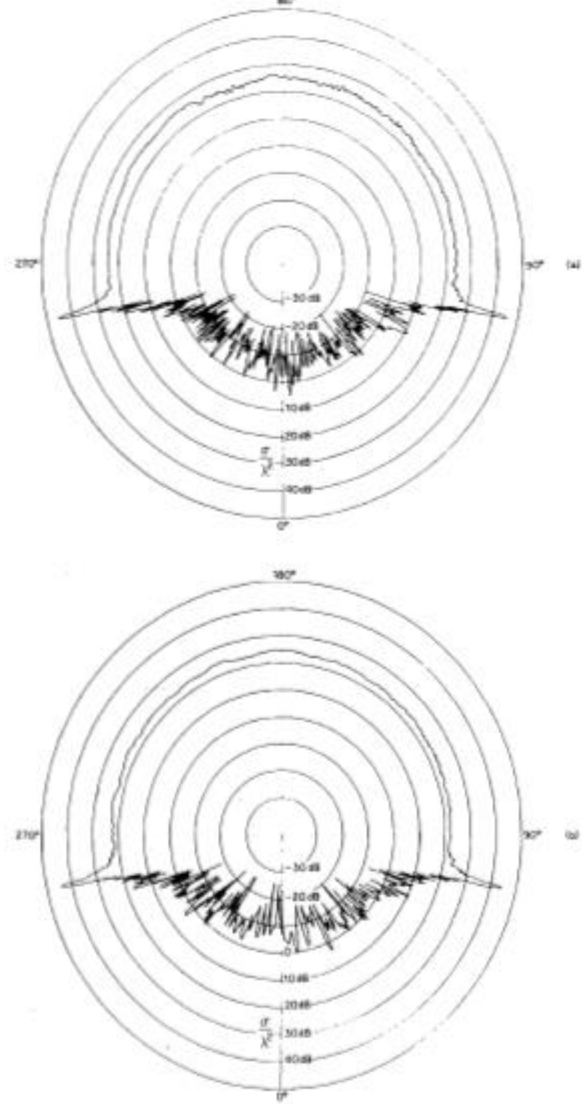
(ii) Surface must be smooth (roughness small compared to wavelength)

(iii) Join between cone and sphere must have a continuous derivative.

(iv) There must be no holes, or protuberances on the surface.

(v) Shaping targets like a cone-sphere is a good method to reduce RCS of targets.

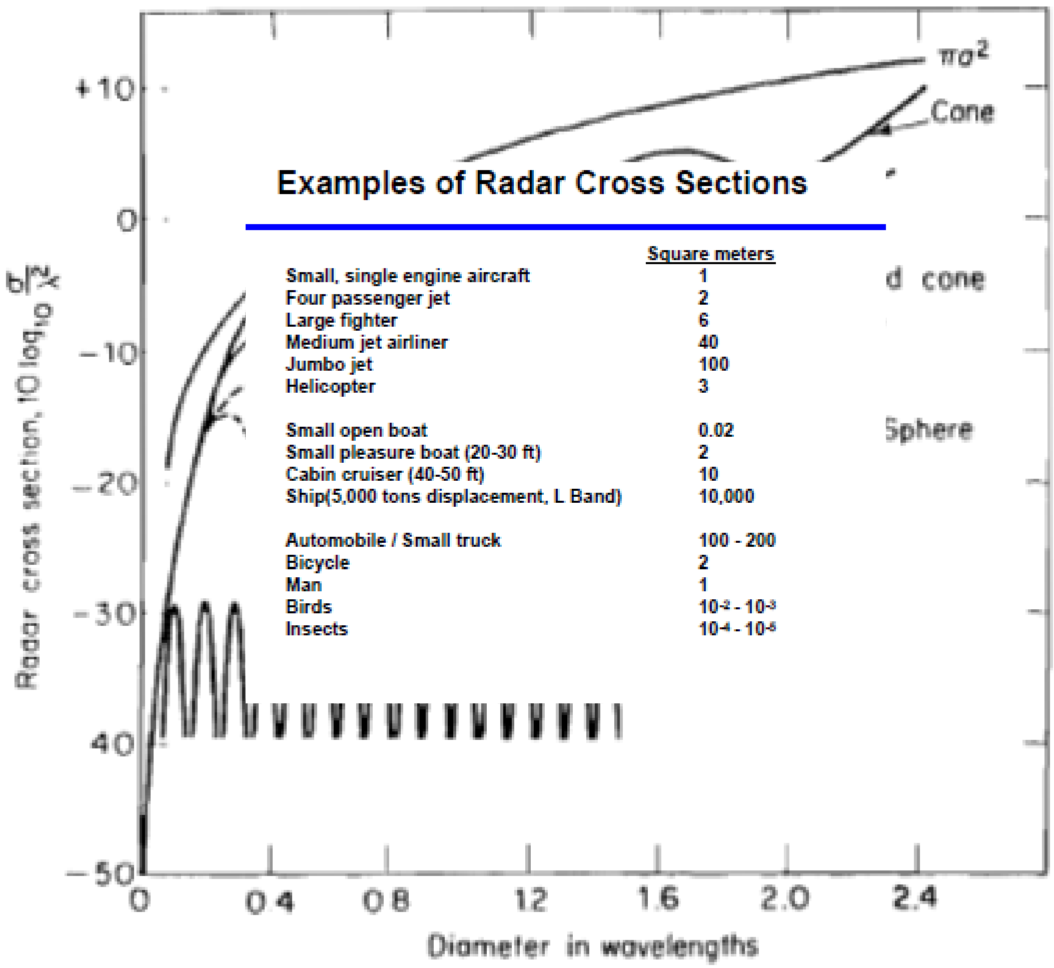
(vi) Carbon-fiber composites are used as material for the target.



Measured radar cross section (σ/λ^2 given in dB) of a large cone-sphere with 12.5° half angle and radius of base = 10.4λ. (*a*) horizontal (perpendicular) polarization, (*b*) vertical (parallel) polarization (*From Pannell et al.*[61])

Radar Cross Section (RCS) (Contd…)

- Comparison



Examples of Radar Cross Sections

	Square meters
Small, single engine aircraft	1
Four passenger jet	2
Large fighter	6
Medium jet airliner	40
Jumbo jet	100
Helicopter	3
Small open boat	0.02
Small pleasure boat (20-30 ft)	2
Cabin cruiser (40-50 ft)	10
Ship(5,000 tons displacement, L Band)	10,000
Automobile / Small truck	100 - 200
Bicycle	2
Man	1
Birds	10^{-2} - 10^{-3}
Insects	10^{-4} - 10^{-5}

Examples of Radar Cross Sections

	Square meters
Small, single engine aircraft	1
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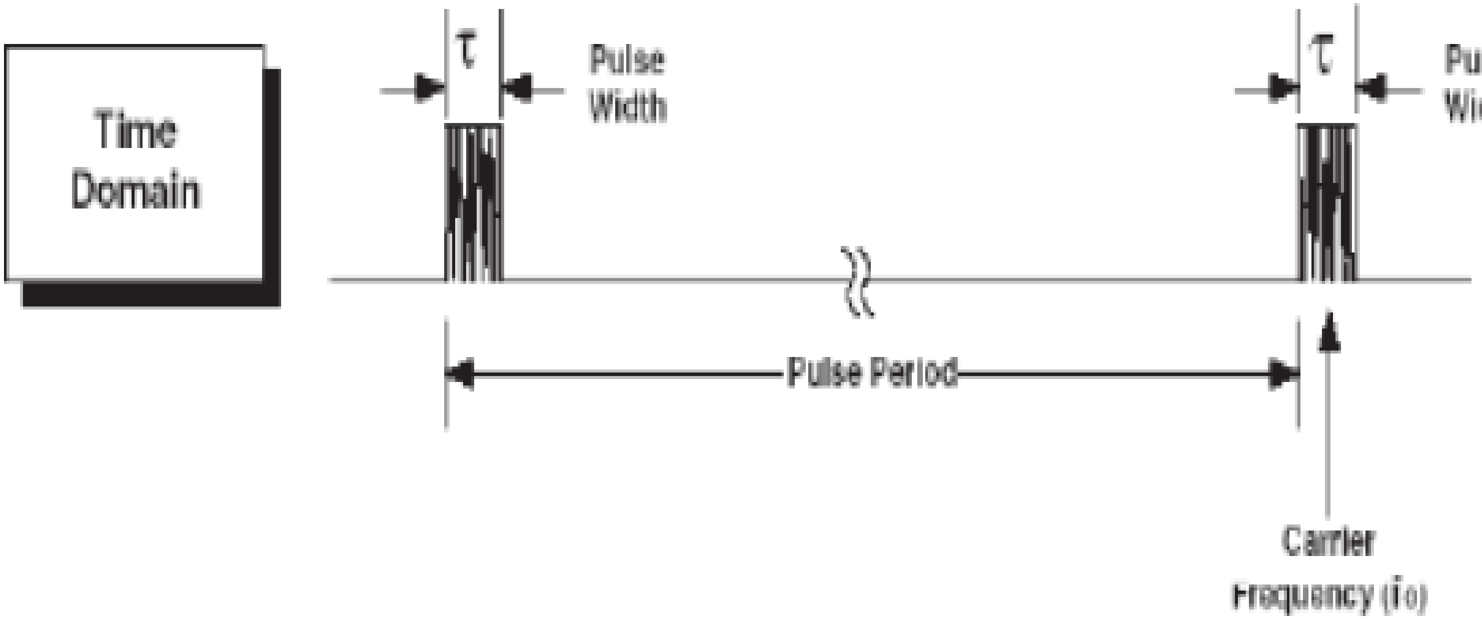
TRANSMITTER POWER

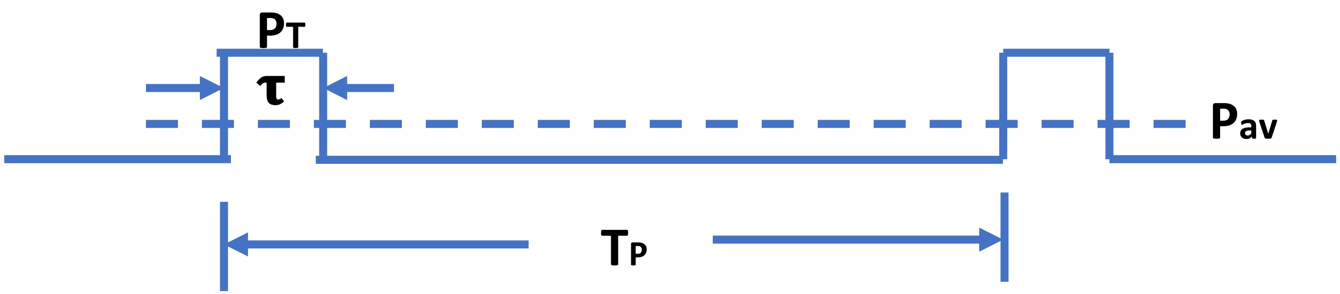
TRANSMITTER POWER

➢



TRANSMITTER POWER

- . (Jntuh) Define and explain transmitted power in Radar equation and express Radar equation in terms of the energy contained in the transmitted power



- P_T = Peak Power ; P_{av} = Average Power ;
- Energy = Power x time
- Energy in Peak power = Energy in Average Power
- $P_T \tau = P_{av} T_P$; So $P_{av} = \frac{P_T \tau}{T_P} = P_T \tau f_p$

TRANSMITTER POWER

- Duty cycle = $\frac{\tau}{T_P} = \frac{P_{av}}{P_T}$ Therefore $P_T = \frac{P_{av}}{\tau \ f_P}$ Eq. No.1
- $R_{max} = \left[\frac{P_T \ G \ A_e \ \sigma \ n \ E_i(n)}{(4\pi)^2 \ K \ T_0 \ B_n \ F_n \ (S/N)_1}\right]^{\frac{1}{4}}$ Eq. No. 2
- $E_i(n)$ = Integration efficiency ; F_n = Receiver noise power
- $(S/N)_1$ = Signal-to- Noise ratio of single pulse

-

TRANSMITTER POWER

- Combining Eq.No.1 & 2
- $R_{max} = \left[\frac{P_{av} \; G \; A_e \; \sigma \; n \; E_i(n)}{(4\,\pi\,)^2 \; K \; T_0 \; (B_n\,\tau) \; F_n \; (S/_N)_1 \; f_p}\right]^{\frac{1}{4}}$ Eq. No. 3
- $(B_n \, \tau)$ = Band width × Pulse width (normally set to 1 in radars)
- Energy = $E_T = P_{av} \, T_P = \frac{P_{av}}{f_P}$ Eq.No. 4
- Substituting Eq. No. 4 in Eq. No. 3 we have
- $R_{max} = \left[\frac{E_T \; G \; A_e \; \sigma \; n \; E_i(n)}{(4\,\pi\,)^2 \; K \; T_0 \; F_n \; (S/_N)_1}\right]^{\frac{1}{4}}$
-

TRANSMITTER POWER.

- Range R_{max} depends on i) E_T Total transmitted energy

  ii) Gain of Antenna

  iii) Aperture area of antenna

  iv) Receiver noise power

- (Jntuh) Explain how the transmitted power affects the range

- $R_{max} = \left[\frac{P_T \; G \; A_e \; \sigma}{(4\pi)^2 \; S_{min}}\right]^{\frac{1}{4}}$

- Considering all factors other than P_T, constant

- $R_{max} = [K \; P_T]^{\frac{1}{4}}$

- $R_{max} \propto [P_T]^{\frac{1}{4}}$

TRANSMITTER POWER

- To get double the range transmitter power has to be increased by 16 times
-

Range	P_T
2 Times	16 Times
3 Times	81 Times
Half	1/16 Times $(1/2)^4$

-

PULSE REPETITION FREQUENCY
&
RANGE AMBIGUITIES

PRF AND RANGE AMBIGUITIES (CONTD…)

- (Jntuh) **Describe how pulse repetition frequency of a radar system controls the range of its detection**

OR

- (Jntuh) **Discus the factors of PRF and range ambiguities**

OR

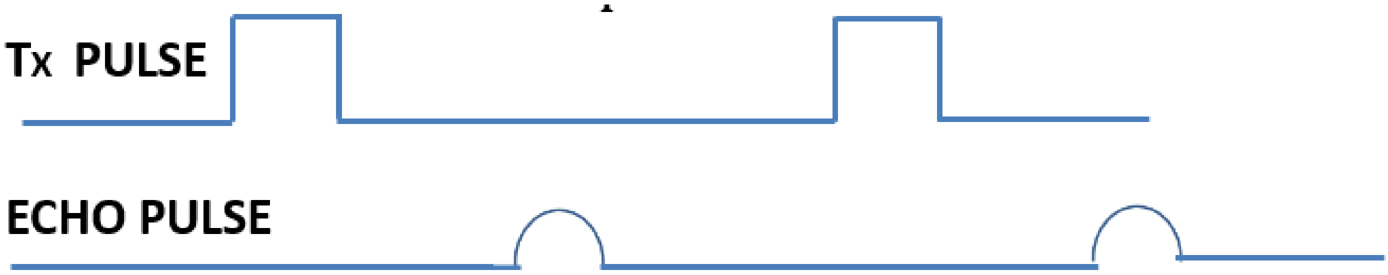
- (Jntuh) **Bring out the restrictions on the selection of Pulse Repetition Frequency in radar operation**

OR

- (Jntuh) **Discuss the factors affecting the PRF and range of a radar**

PRF AND RANGE AMBIGUITIES (CONTD…)

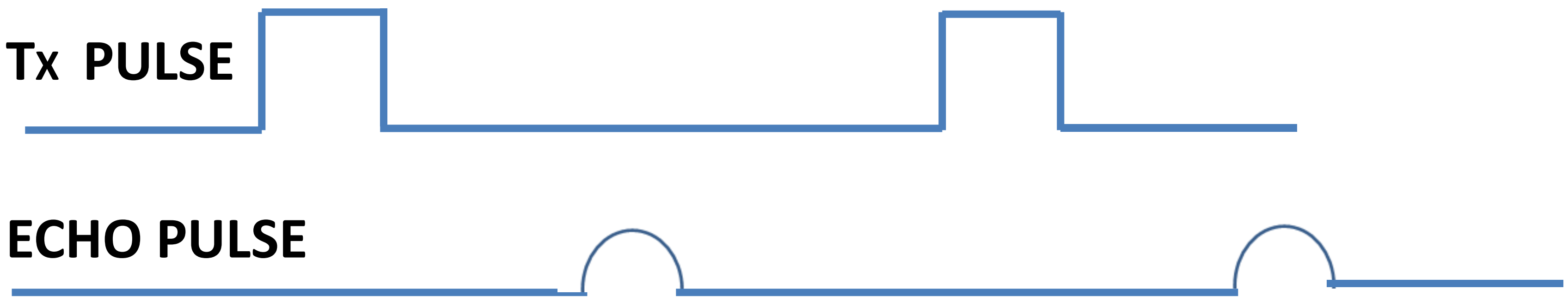
- Max Range = $\frac{C\, T_P}{2} = \frac{C}{2\ f_p}$



- Range Ambiguity:
- Once the transmitted pulse is sent into space , sufficient length of time must be allowed for the Echo to return, before the next pulse is sent

PRF AND RANGE AMBIGUITIES (CONTD…)

•



- If PRF is high (T_P is low), Echo from the target may arrive after the transmission of next pulse . This causes confusion and is called Range Ambiguity

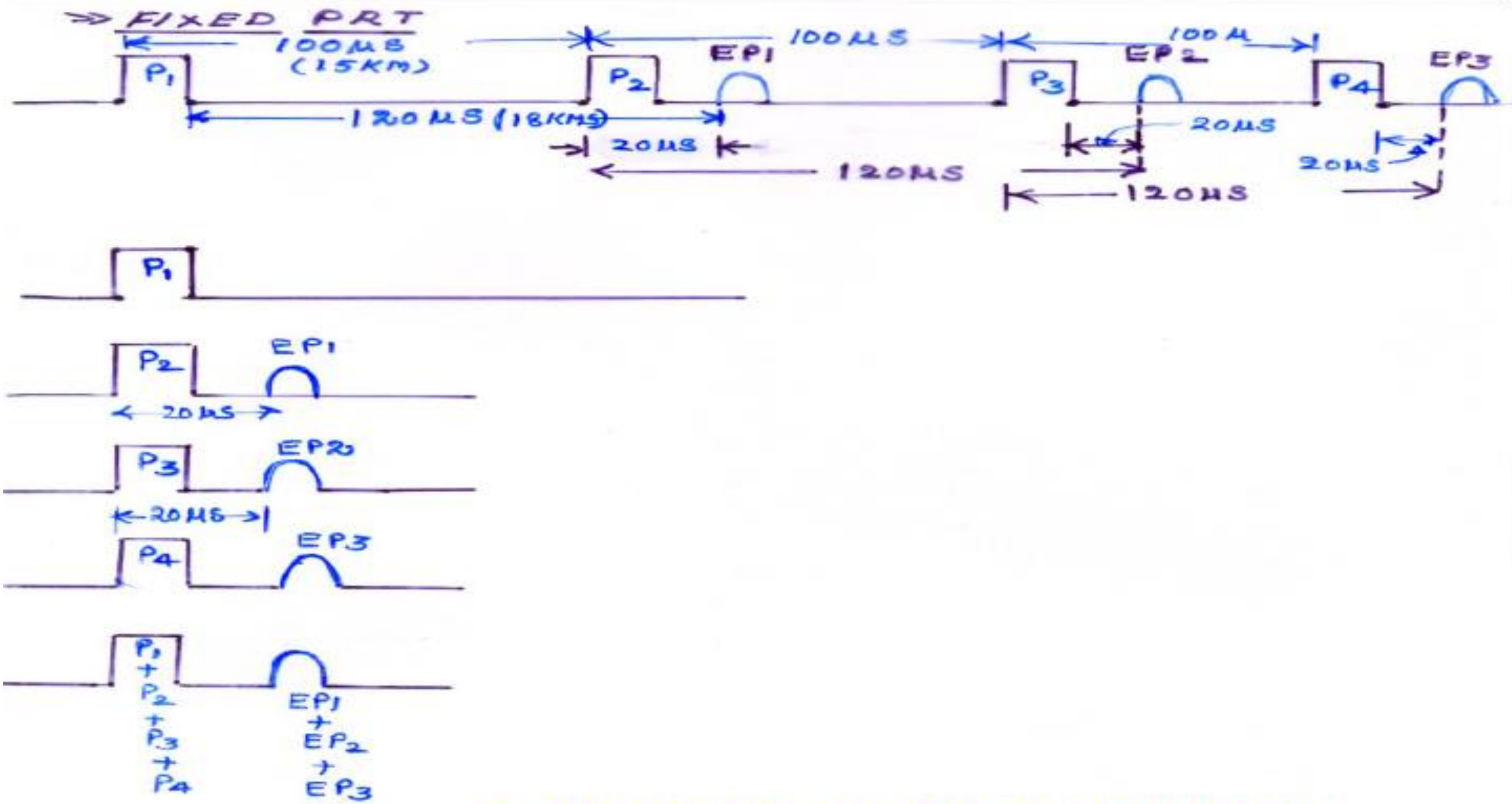
PRF AND RANGE AMBIGUITIES (CONTD…)

- If PRF is high (T_P is low), Echo from the target may arrive after the transmission of next pulse . This causes confusion and is called Range Ambiguity
- Echoes that arrive after the transmission of second pulse are called 2nd time around Echoes

➢ Multiple Time Around Echoes:

- If f_P is very high, R_{max} is low, there is likelihood of receiving target echoes from targets beyond R_{max}
- Echo signals received after the Pulse Repetition Interval ($1/f_P$) are called multiple-time around echoes. They result in confusing Range measurements.

SECOND TIME AROUND ECHOES –RANGE AMBIGUITIES

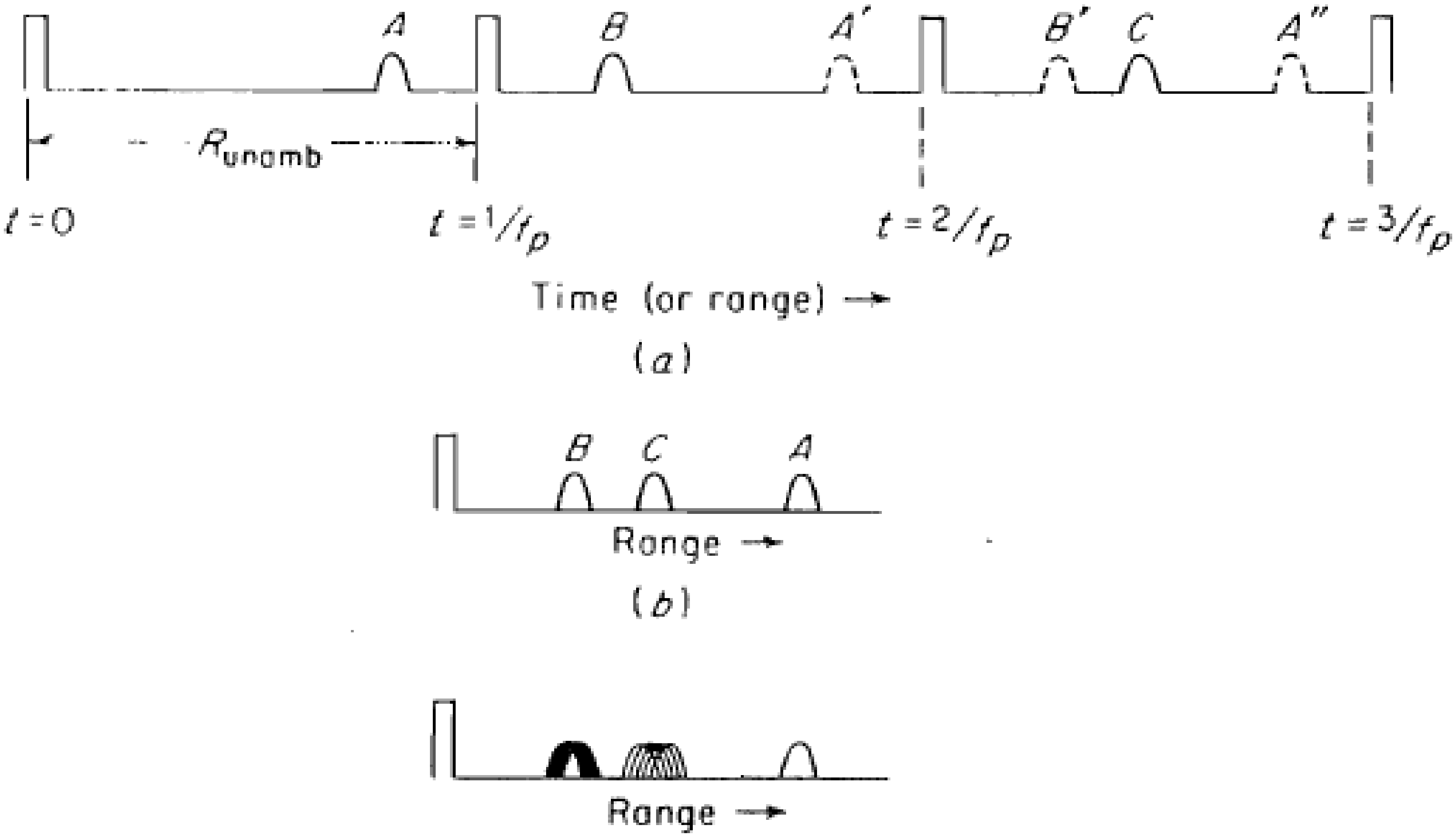


> DETECTION OF RANGE AMBIGUITY NOT POSSIBLE

PRF AND RANGE AMBIGUITIES (CONTD…)

- **Multiple Time Around Echoes:**
- If f_P is very high, R_{max} is low, there is likelihood of receiving target echoes from targets beyond R_{max}
- Echo signals received after the Pulse Repetition Interval ($1/f_P$) are called multiple-time around echoes. They result in confusing Range measurements.

PRF AND RANGE AMBIGUITIES (CONTD…)



Multiple Time Around Echoes

PRF AND RANGE AMBIGUITIES (CONTD…)

- Target 'A'located within unambiguous Range R_{max}
- Target 'B' located beyond R_{max} but less than $2R_{max}$
- Target 'C' located beyond $2R_{max}$ but less than $3\ R_{max}$

- On the 'A' scope only the Range measured of Target A is correct but for target 'B' and 'C'. The range is erroneous (ambiguous).

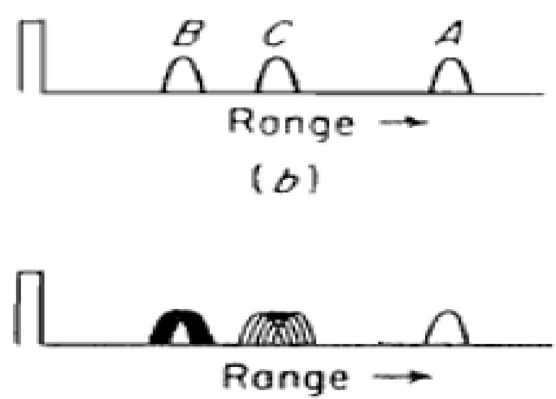
PRF AND RANGE AMBIGUITIES (CONTD…)

- The multiple time around Echoes can be found out from the Unambiguous Echoes by using a varying PRF

➢ **Distinguishing Multiple –Time - Around echoes**

- Operate the Radar with varying PRF
- Echo signal from 'A' (unambiguous range) appear

at the same place on the sweep irrespective of change in PRF. Targets echoes 'B' & 'C' will spread as shown in Figure

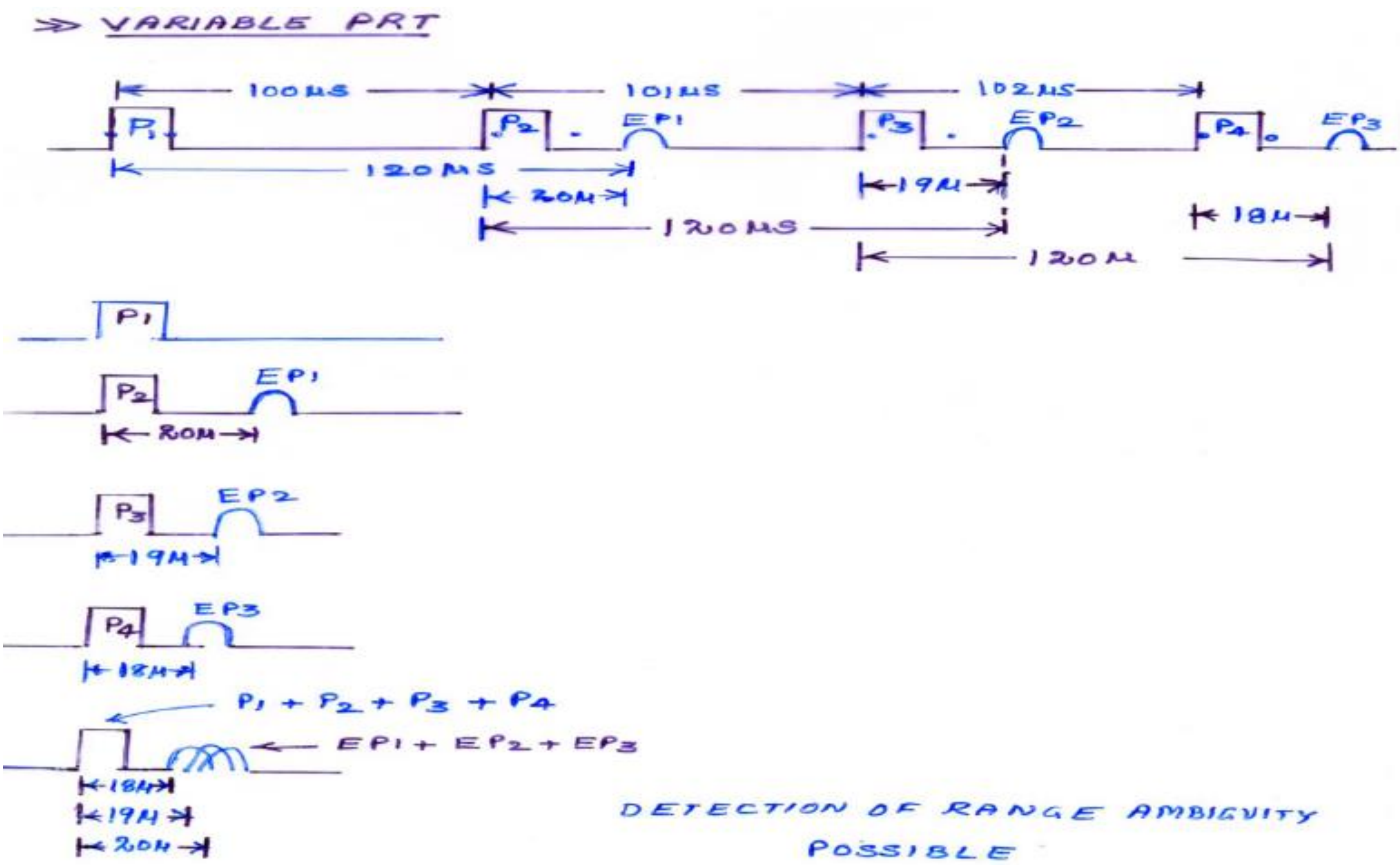


PRF AND RANGE AMBIGUITIES (CONTD…)

➢ Distinguishing Multiple –Time - Around echoes

- Second Time targets need 2 separate PRFs in order to be distinguished.
- Other methods to resolve Range Ambiguities are by changing from pulse to pulse (i) Amplitude (ii)Pulse width (iii) Polarization

SECOND TIME AROUND ECHOES –RANGE AMBIGUITES (CONTD…)



CONTINUED IN RADAR 1 G

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SYSTEM LOSSES

SYSTEM LOSSES (Contd…)

- Losses are inevitable in any practical system
- Losses reduces the signal-to-noise ratio
- Radar Equation taking system Losses into consideration becomes
- $R_{max}{}^{4} = \dfrac{P_{av}\ G\ A_e\ \sigma\ n\ E_{i(n)}}{(4\pi)^2\ K\ T_O\ F_N\ (\beta\tau)\, f_p\ \left({}^{S}/_{N}\right)_1\ L_S}$
- L_s = System Losses (greater than unity)
- Loss is a number greater than unity
- Efficiency is a number less than unity
- Loss = $\dfrac{1}{\text{Efficiency}}$

SYSTEM LOSSES (Contd...)

Radar Losses

- Predictable
- Less Predictable

Less Predictable Loss

- Operator Loss
- Field Degradation Loss
- Nonideal equip-ment loss

Predictable Loss

- Plumbing loss
- Antenna loss
- Limiting loss
- Collapsing loss

SYSTEM LOSSES (Contd…)

➢ Less Predictable Loss:

- Estimation of this loss depends on experimentation and earlier observation.
- **Categories:** (i) Operator loss (ii) Field Degradation Loss (iii) Non ideal Equipment Loss

➢ Predictable Loss:

- This loss is predicted if system configuration is known
- **Categories** : (i) Plumbing loss (ii) Antenna loss iii)Limiting Loss iv) Collapsing Loss

PREDICTABLE LOSSES

PREDICTABLE LOSSES

A. Plumbing Loss:

- Subdivided into following losses

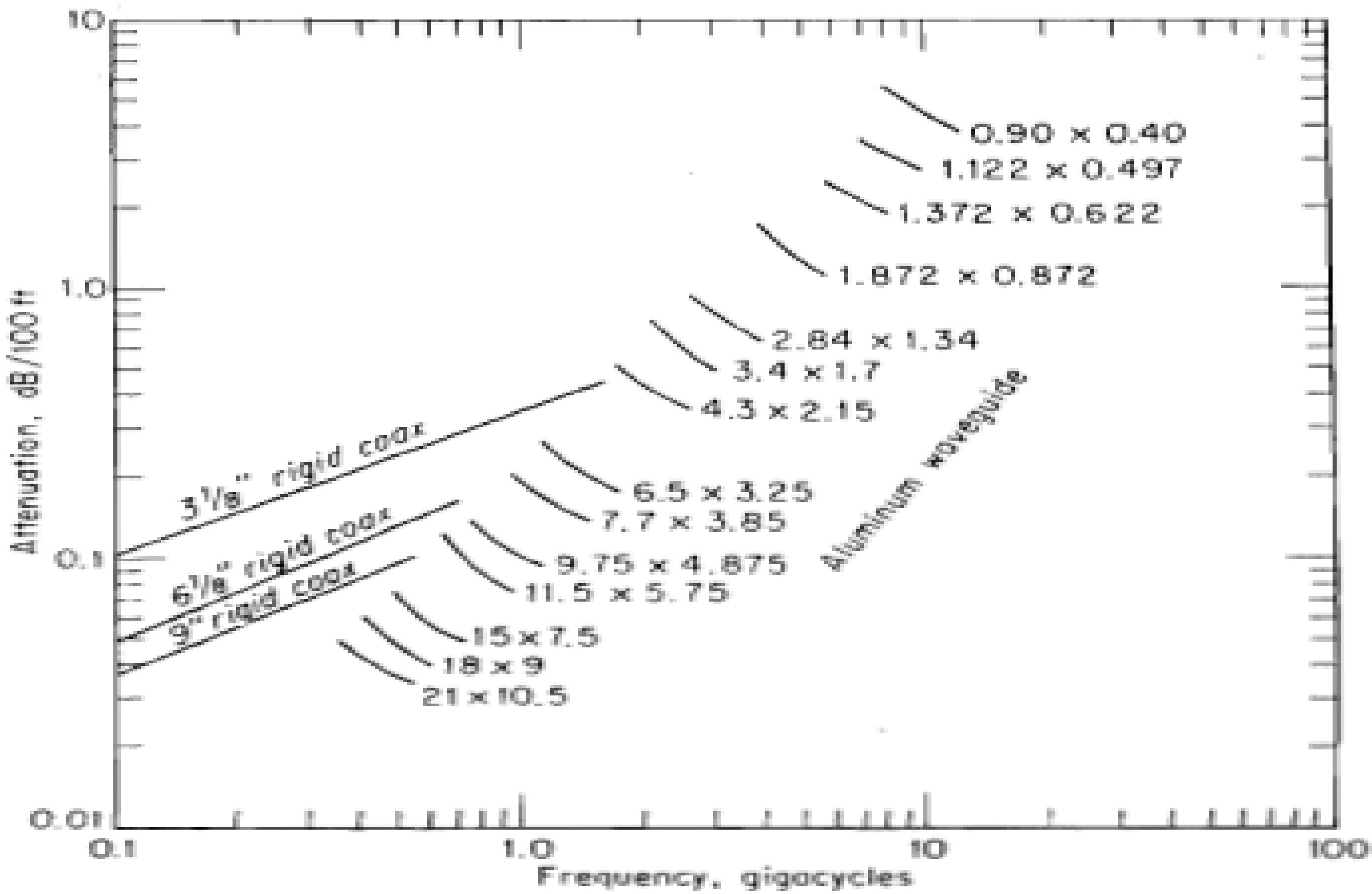
1. Transmission loss
2. Connector loss
3. Bend loss
4. Rotary Joint loss
5. Duplexer loss
6. Wave Guide Shutter

1. Transmission Loss:

- Tx and Rx are connected to Antenna by wave-guides or coaxial cables called Transmission lines
- Losses are normally expressed in Decibels
- At Lower frequencies the transmission loss is less

PLUMBING LOSS (CONTD…)

I. Transmission Loss (Contd…)



PLUMBING LOSS (Contd…)

- **2. Connector Loss:**
- **3. Bend Loss:** Waveguides have bends
- **4. Rotary Joint Loss:** Power from the fixed transmitter Amplifier is connected to Antenna which is mechanically rotating
- **5.Duplexer Loss:** Duplexer Loss is different when transmitting from when receiving
- **6. Waveguide Shutter:** Protects transmitter when not transmitting from adjacent radar transmissions
- Total loss = 2 x one way Loss (since same components are used for transmission & receiving)

PLUMBING LOSS (CONTD…)

- **Plumbing Losses in a 300 MHZ Radar**

(1) Waveguide transmission (2way) = 1.0 dB

(2) Loss due to connectors & bends = 0.5 dB

(3) Rotary joint Loss = 0.4 dB

(4) Duplexer Loss = 1.5 dB

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Total Plumbing Loss = 3.4 dB

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ANTENNA LOSS (CONTD…)

- **B. Antenna Loss:**
  1. Beam Shape loss
  2. Scanning loss
  3. Radome loss
  4. Polarisation loss
  5. Squint loss

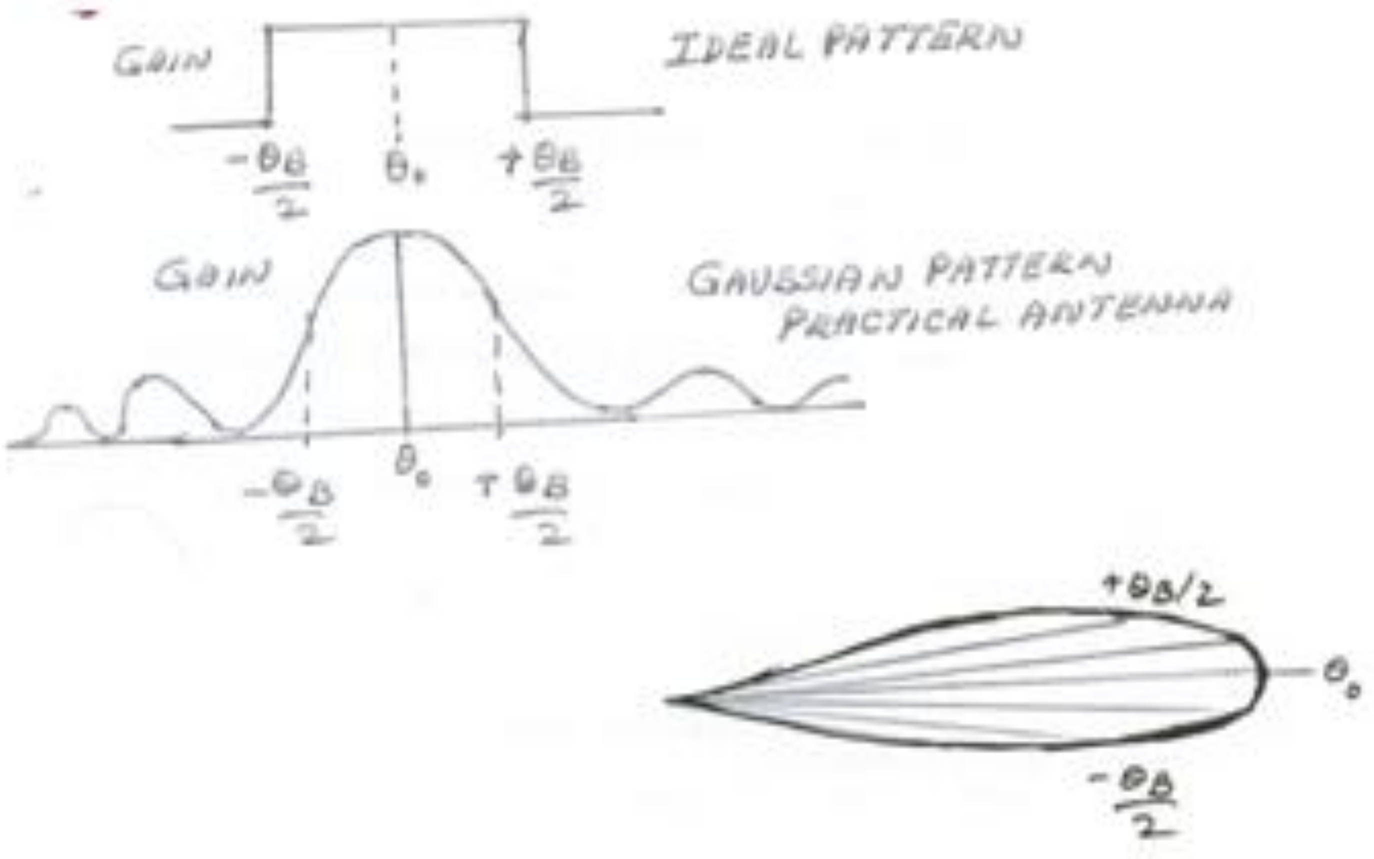
ANTENNA LOSS (CONTD…)

1. Beam Shape Loss :

- In Radar Equation
- $S_{min} = \frac{P_T \; G^2 \; \sigma \; \lambda^2}{(4\,\pi)^3 \; {R_{max}}^4}$

- In the above equation Antenna Gain is assumed constant. But in practice this is not true
- Beam sweeps through target, received pulses amplitude will be modulated by Beam pattern.
- In the above equation for the value G (i) Employ Average value of various gains at different angles or (ii) Maximum gain occurring at centre of the beam

BEAM SHAPE LOSS



BEAM SHAPE LOSS (CONTD…)

- Because of the above assumption, beam shape loss takes place and is given by the equation
- Beam shape Loss = $\dfrac{n}{1 + 2^{(n-1)} \sum_{K=1}^{(n-1)/2} \exp\left(-5.55 \dfrac{K^2}{{n_B}^2}\right)}$

Where n = no. of pulses integrated

n_B = no. of pulses received within half power Beam width

ANTENNA LOSS (CONTD…)

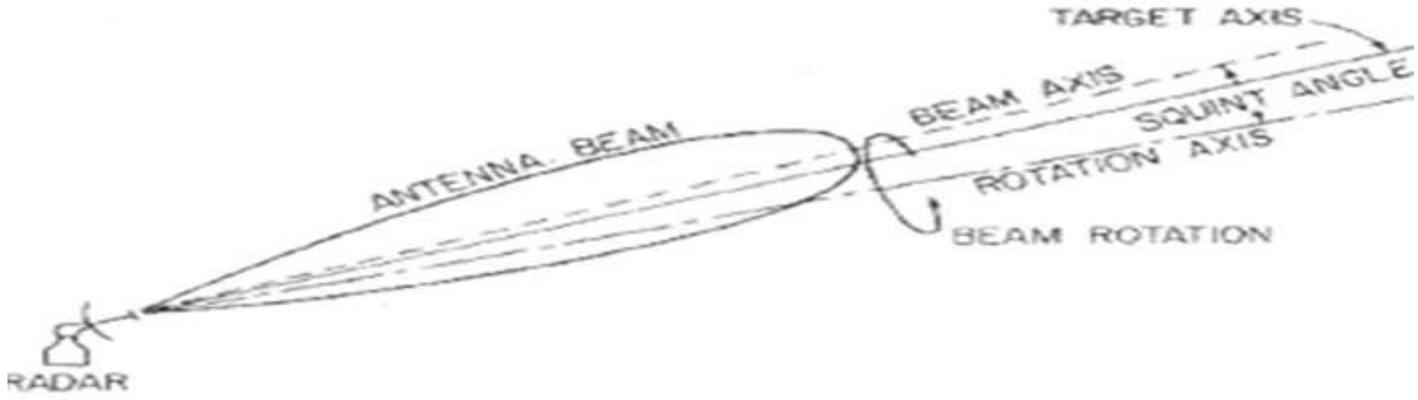
- **2. Scanning Loss:**
- If Antenna Rotation is fast, Gain of Antenna for the transmitted pulse is different for the received pulse. This introduces loss and is called as Scanning Loss.
- Scanning Loss can be considerable for fast scan antennas, and for those with long interval between pulses designed for viewing extraterrestrial objects.

ANTENNA LOSS (CONTD…)

- **3. Radome Loss:**
- Radome is used to protect Antenna from extremes of weather conditions ie during storms, rain etc.,. Radome is transparent to Radar frequencies.
- Radome introduces loss and depends on frequency
- Approximate loss at L to X band is 1.2 dB

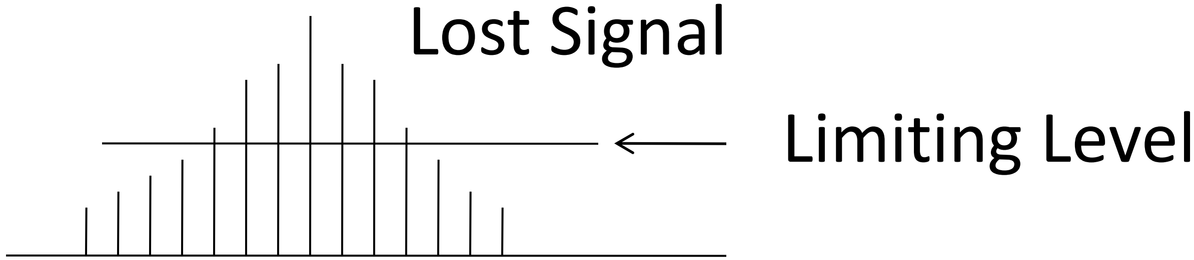
ANTENNA LOSS (CONTD…)

- **4. Polarization Loss:**
- There is a mismatch between transmission and reception. This is due to Polarization.
- **5. Squint Loss:**
- In conical scan Radars, Antenna Beam is squinted from tracking axis. This introduces loss called squint loss



PREDICTABLE LOSSES (CONTD…)

C. Limiting Loss



- Limiting lowers the probability of detection
- CRT displays or B scope have limited dynamic range and limits the received echoes. This lowers the power processed in the receiver.

LIMITING LOSS (CONTD…)

- Limiting Ratio = $\frac{\text{Video limit level}}{\text{RMS Noise level}}$

- If the number of pulses integrated are large and limiting ratio is 2 or 3 Limiting loss is only a fraction of Decibel
- In case of Band Pass limiters for small S/N ratio

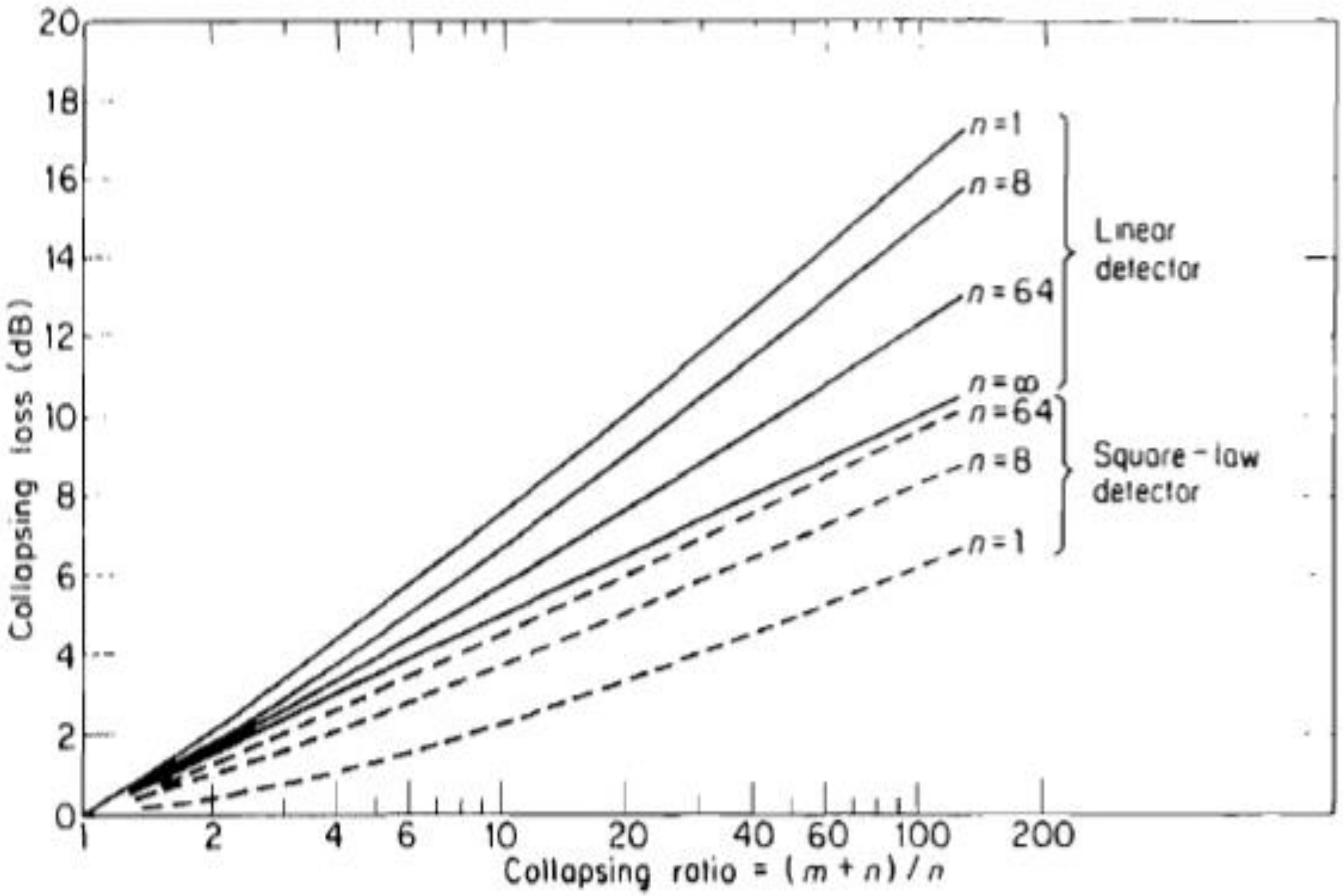
  the reduction in S/N is about 1 dB

PREDICTABLE LOSSES (CONTD…)

D. Collapsing Loss:

- Radar integrates additional noise samples along with the wanted signal-plus-noise pulses. The added noise results in a degradation called collapsing loss
- C-scope displays elevation Vs azimuth angle, but collapses the range information which means for a particular elevation and azimuth angle samples are taken throughout the range. This results in noise energy being sampled in all the ranges.
- In 3 D Radars (range, azimuth & elevation) 3 D information is displayed on 2 D display (PPI). This results in collapsing of one coordinate.

COLLAPSING LOSS VS COLLAPSING RATIO



COLLAPSING LOSS (CONTD…)

- Collapsing loss =
- $L_{i(m,n)} = \frac{L_{i(m,n)}}{L_{i(n)}}$
- where 'm' = no. of noise pulses
- 'n' = n signal-plus-noise pulses
- Assume n = 10 signal-plus-noise pulses are integrated with m = 30 noise pulses.
- For a of $P_d = 0.90$ and $n_f = 10^8$, $L_i(40) = 3.5$ dB
- L_i (10) = 1.7 dB so that L_i (m, n) collapsing loss = 1.8 dB

LESS PREDICTABLE LOSSES

LESS PREDICTABLE LOSSES

➢ 1. Operator Loss:

- When operators are untrained, distracted, tired, or overloaded the performance degrades. This can be assigned to operator loss.
- If ρ_0 is operator efficiency factor, P_D is single scan probability of detection

$$\rho_0 = 0.7\,(P_d)^2$$

LESS PREDICTABLE LOSSES (CONTD…)

➢ 2. **Field Degradation Loss:**

- Field conditions under which Radar operates degrades the performance of Radar compared to when it is operated in a laboratory.
- In addition field conditions vary vastly
- Factors which contribute to performance degradation are poor tuning, water in transmission lines, deterioration in receiver noise figure, loose cable connections, extreme cold, fog, rain, heat, dust etc.

FIELD DEGRADATION LOSS (CONTD…)

- All the above are grouped into a loss factor called Field Degradation Loss.
- These losses can be reduced to certain extent by monitoring the degradation through built-in-automatic equipment and making corrections and tuning at regular intervals.
- Preventing maintenance reduces this loss.

LESS PREDICTABLE LOSS (CONTD…)

- **Non Ideal Equipment Loss:**
  - 1.Transmitting Power P_T used in Radar Range equation vary from one Transmitting tube to another, as they are not uniform in power output or quality
  - 2. P_T degrades with time due to aging
  - 3. Variation in receiver noise figure over time are to be expected
  - 4. If receiver is not exactly a matched filter loss to S/N will occur (1 dB)
- To take into consideration the above Non Ideal Equipment loss is introduced

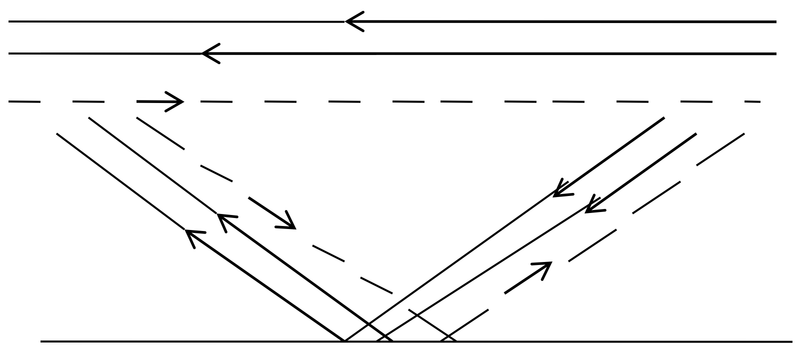
PROPAGATION EFFECTS

PROPAGATION EFFECTS

- The medium under which the radar waves propagate can have significant effect on radar performance.
- Sometimes the propagation losses are very significant and contribute to abnormal radar performance.
- Attenuation of radar waves from rain often limits the performance of radar.
- Decreasing density of atmosphere with increase in altitude results in bending of wave. This increases the path length of LOS.

PROPAGATION EFFECTS (CONTD…)

- These losses are different at various frequencies and generally lower at low frequencies.
- **GROUND PLANE LOSS**



- Presence of earth’s surface cause major modification of the coverage.
- Energy directly travels to target from radar antenna. There can be energy that travels to the target after reflection from ground.

PROPAGATION EFFECTS (CONTD…)

- Direct and ground reflected waves interfere at target either destructively or constructively
- This may create nulls or reinforcements
- Performance degradation because of this is grouped into Ground Plane Loss.

MODIFICATION OF RADAR RANGE DUE TO LOSSES

MODIFICATION OF RADAR RANGE DUE TO LOSSES

- $$R_{max}{}^{4} = \frac{P_{av}\ G\ A_e\ \rho_a\ \sigma\ n\ E_{i(n)}}{(4\,\pi)^2\ K\ T_O\ F_n\ (\beta\,\tau)\,f_p\ (S/N)_1\ L_S}$$

where $\boldsymbol{\rho_a}$ = Antenna efficiency

L_s = Losses grouped together

➢ Radar Performance Figure:

This is a figure of merit used to express the relative performance of radar

$$= \frac{\text{Pulse power of radar transmitter}}{\text{Minimum detectable signal power}}$$

SURVEILLANCE RADAR RANGE EQUATION

- Radar equation given earlier, applies to a radar that dwells on a target for n pulses.
- Search and surveillance Radar is required to search a specified volume of space within a specified time
- If Ω = Angular region say 360^0 in azimuth, 30^0 in elevation

  t_o = time on target = n/f_p

  Ω = Solid angular beam width $(\theta_a \times \theta_e)$

  θ_a= azimuth beam width θ_e= Elevation Beam width

  Scan time = $t_o = \frac{t_o\ \Omega}{\Omega_o}$

  $G = \frac{4\ \pi}{\Omega_o}$

Surveillance Radar Range Equation (Contd…)

- $$R_{max}^{4} = \frac{P_{av}\ G\ A_e\ \rho_a\ \sigma\ n\ E_{i(n)}}{(4\pi)^2\ K\ T_O\ F_n\ (\beta\tau)\ f_p\ (S/N)_1\ L_S}$$

➤ The important parameters for a search Radar are

(i) Average Power

(ii) Antenna effective aperture

$$R_{max}^{4} = \frac{P_{av}\ A_e\ \sigma\ E_{i(n)}\ t_o}{4\ \pi\ K\ T_O\ F_n\ (S/N)_1\ L_o\ \Omega}$$

PROBLEMS

PROBLEM 1:

The Bandwidth of I.F. Amplifier in a Radar Receiver is 1 MHZ. If the Threshold to noise ratio is 12.8 dB. Determine the False Alarm Time.

T_{fa} = False Alarm Time

$$T_{fa} = \frac{1}{BIF} \exp \frac{V_T^2}{2\ \varphi_O}$$

where $B_{IF} = 1X\ 10^6$ HZ

Threshold to Noise Ratio = 12.8 dB

$$10 \log_{10} \frac{V_T^2}{2\ \varphi_O} = 12.8 \text{ dB}$$

- $\frac{V_T^2}{2\ \varphi_0}$ =Antilog$_{10}$ $\frac{12.8}{10} = 19.05$
- $T_{fa} = \frac{1}{1 \times 10^6}\ e^{19.05} = \frac{187633284.2}{10^6} = 187.6\ \text{sec}$

PROBLEM – 2:

The PRF of envelope of Noise Voltage is

$P(R) = \frac{R}{b} \exp \frac{-R^2}{2b}$ for $R \geq 0$. If P_{fa} needed is $\leq 10^{-5}$ Determine Threshold Level.

- $P(R) = \frac{R}{b} \exp \frac{-R^2}{2b}$
- $P(R) = \frac{R}{\varphi_0} \exp\left[-\frac{{V_T}^2}{2\ \varphi_o}\right]$
- Where $b = \varphi_0$
- $P_{fa} = 10^{-5} = \exp \frac{-{V_T}^2}{2\ \varphi_o}$

Taking Anti (Natural) Logarithms.

$$-5 \text{ Log } e^{10} = -\frac{{V_T}^2}{2\ \varphi_o}$$

$$-5 \times 2.3026 = -11.5 = -\frac{{V_T}^2}{2\ \varphi_o}$$

$$V_T^2 = 11.5 \times 2\ \psi_0$$

$$V_T = \sqrt{23}\ \sqrt{\varphi_o} = 4.8\sqrt{\varphi_o}$$

END OF UNIT 1